Continuous Sliding Mode Controller Design for Inverted Pendulum System

Saurabh Saini
Department of Instrumentation
USIC, Kurukshetra University
Kurukshetra, India
E-mail: saurabh.ece24@gmail.com

Bhanu Pratap
Department of Electrical Engineering
National Institute of Technology,
Kurukshetra, India
E-mail: bhanumnnit@gmail.com

Surinder Singh
Department of Instrumentation
USIC, Kurukshetra University
Kurukshetra, India
E-mail: surinder_kkr2003@rediffmail.com

***Abstract*—This paper presents a non linear robust controller for inverted pendulum system (IPS) using continuous sliding mode control (CSMC) technique. The CSMC is obtained by replacing the discontinuous element of integral sliding mode control (ISMC) approach. The super twisting control algorithm is used as a disturbance estimator which rejects the effect of disturbances. The undesirable chattering effect due to discontinuous term present in conventional ISMC has been completely removed using proposed continuous approach. Finally, the superiority of proposed control scheme has been demonstrated throw simulation results to depict its effectiveness.**

***Keywords*— Continuous sliding mode control; inverted pendulum; robustness; super twisting algorithm, uncertainty.**

I. INTRODUCTION

The stabilization of inverted pendulum system using robust control techniques have been well known among instrumentation and control engineers and researchers due to its varied application such as disturbance rejection, order reduction and strong robustness against parametric uncertainties. Sliding mode control (SMC) has been widely used in the field of spacecraft control, aircraft control, machine control, process control, power control, motion control, robot control, vehicle control and the recently developed network control and congestion control [1]. Robust controllers are the key element to balance the system affected by disturbances, noise, shock waves and stabilize it.

The design of the robust controllers for IPS is the hot issue in the control field. There are many control laws have been proposed such as proportional-integral-derivative (PID) control, linear quadratic regulator (LQR), fuzzy logic control (FLC), adaptive control law, artificial neural network (ANN) etc. SMC has many advantages than traditional one. It can be applied to linear/nonlinear systems, continuous/discrete systems [2]-[6].

SMC is a type of saturation/relay control with high frequency switching on the switching surface [1]-[3]. Thus, an ideal SMC should be infinitely fast but it is difficult for practical SMC due to space, time, inertia and hysteresis delays. In SMC, there is an inevitably occurs as a chattering problem during sliding mode phase. This chattering is removed by using saturation function but at the cost of robustness [7]-[9]. The sliding mode control approach consist of two phases: (i) reaching phase, (ii) sliding phase. For the implementation of SMC based controllers, there are two steps: (i) selection of the switching surface such that the desired close loop performance can be achieved, (ii) designing of robust control effort law that forces the state trajectories towards the switching surface and stay thereafter [8]-[11]. The higher order system is converted into lower order system in sliding mode approach. It permits control algorithm which is simple for implementation and robust in nature. Sliding mode controllers provide the better control performance without correlation to the system parameters and external disturbances [12]-[15]. Super twisting and higher order sliding mode observer based controller for the position control of industrial emulator is illustrated in [16].

In this paper, a continuous sliding mode controller is proposed for the IPS to stabilize the unstable equilibrium points globally. The chattering effect has been discarded completely using proposed continuous approach. The remainder of this paper is organized as follow. In section second II, the mathematical modeling of the inverted pendulum system and brief introduction about Lie derivative is given Section III problem statement is formulated. Conventional and continuous sliding mode controllers are designed in Section IV and V respectively. Section VI demonstrates the simulation results of the proposed robust controllers. The concluding remarks have been illustrated the last section.

II. PRELIMINARIES

II.A. Mathematical Modelling of inverted pendulum system:

The inverted pendulum is taken into fourth order, multivariable unstable single-input-single-output (SISO) system [2]. It consist a cart with an attached pole and the cart is able to move to maintain the pole in the upright position. A linear motor is used for the motion of the cart for balancing the IPS. The main objective is to balance the bob in upright position and prevent it from falling. The inverted pendulum is shown in Fig.1. The inverted pendulum system consists of a cart, a pole and a motor. Under normal conditions the pole of the pendulum would be in downward position. If the pole is held upright it would have the tendency to swing the downwards hence the supply voltage of the rotor works as a

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controller and move the cart to maintain the upright position of the pole and thus forming a robust IPS.

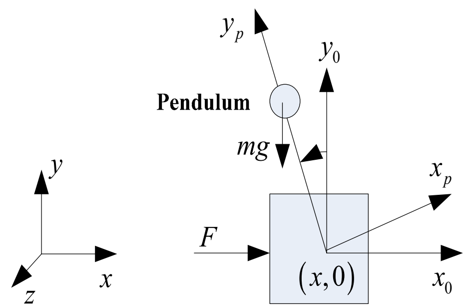

Fig.1 Inverted pendulum system [2]

The dynamical modeling of the inverted pendulum system is derived using Lagrangian formulation is given by,

$$(m+M)\ddot{x}-ml\left(\ddot{\theta}\cos(\theta)-\dot{\theta}^2\sin(\theta)\right)=F \quad (1)$$

$$\left(ml^2+I_P\right)\ddot{\theta}-ml\cos(\theta)\ddot{x}-mgl\sin(\theta)=0 \quad (2)$$

where x=distance travelled by the cart, θ=angle of the pendulum, and F=applied force on the wagon.

The complete dynamical behavior of the inverted pendulum system in the form of differential equations is as follow,

$$\ddot{x}=\frac{F.a-\left(ac\sin(\theta)\right)\dot{\theta}^2+c^2g\cos(\theta)\sin(\theta)}{\left(ab-c^2\cos^2(\theta)\right)} \quad (3)$$

$$\ddot{\theta}=\frac{c\cos(\theta)\ddot{x}+cg\sin(\theta)}{a} \quad (4)$$

where, $a=\left(ml^2+I_P\right)$, $b=(m+M)$, and $c=ml$.

The IPS (3), (4) can be expressed in form of state space model given by,

$$\dot{x}_1=x_2\equiv f_1(x) \quad (5a)$$

$$\left.\begin{aligned}\dot{x}_2&=\frac{c\sin(x_3)\left(cg\cos(x_3)-ax_4^2\right)}{ab-c^2\cos^2(x_3)}+\frac{a}{ab-c^2\cos^2(x_3)}u\\&\equiv f_2(x)+g_2(x)u\end{aligned}\right\} \quad (5b)$$

$$\dot{x}_3=x_4\equiv f_3(x) \quad (5c)$$

$$\left.\begin{aligned}\dot{x}_4&=\frac{c^2\sin(x_3)\cos(x_3)\left(cg\cos(x_3)-ax_4^2\right)}{a\left(ab-c^2\cos^2(x_3)\right)}+\frac{cg\sin(x_3)}{a}\\&+\frac{c\cos(x_3)}{ab-c^2\cos^2(x_3)}u\equiv f_4(x)+g_4(x)u\end{aligned}\right\} \quad (5d)$$

$$y=x_3 \quad (5e)$$

where, $X=\left[x\quad\dot{x}\quad\theta\quad\dot{\theta}\right]=\left[x_1\quad x_2\quad x_3\quad x_4\right]$= state vector, u = input and y = output. The system parameters are given in Table I.

Table I Parameters of the inverted pendulum system

Parameters	Values
M = Mass of the wagon	2 kg
m = Pendulum mass	0.026 kg
I_p = Moment of inertia of pendulum	$0.000362\ \mathrm{kg-m^2}$
l = Half-length of pendulum	0.1 m
g = Acceleration due to gravity	$9.8\ \mathrm{m/sec^2}$

II.B. Lie Derivatives

Consider the nonlinear SISO system described by the state equation,

$$\dot{\bar{x}}=\bar{f}(\bar{x})+\bar{g}(\bar{x})\bar{u}$$
$$\bar{y}=\bar{h}(\bar{x})$$

where $\bar{f}$, $\bar{g}$, and $\bar{h}$ are sufficiently smooth in a domain $\bar{D}\in\Re^n$. The mappings $\bar{f}:\bar{D}\to\Re^n$ and $\bar{g}:\bar{D}\to\Re^n$ are called vector fields on $\bar{D}$ and $\bar{x}$ is an $n-$dimensional state vector that is assumed to be measurable, $\bar{u}$ is a scalar input, and $\bar{y}$ is a scalar output. The derivative $\dot{\bar{y}}$ is given by [17],

$$\dot{\bar{y}}=\frac{\partial\bar{h}}{\partial\bar{x}}\left[\bar{f}(\bar{x})+\bar{g}(\bar{x})\bar{u}\right]=L_{\bar{f}}\bar{h}(\bar{x})+L_{\bar{g}}\bar{h}(\bar{x})\bar{u}$$

where the Lie derivative of a scalar function $\bar{h}(\bar{x})$ with respect to vector function $\bar{f}(\bar{x})$ is given by [17] ,

$$L_{\bar{f}}\bar{h}(\bar{x})=\frac{\partial\bar{h}}{\partial\bar{x}}\bar{f}(\bar{x})$$

This is the familiar notion of the derivative of $\bar{h}$ along the trajectories of the system $\dot{\bar{x}}=\bar{f}(\bar{x})$.

II.C. Normal Form Modelling of The Inverted Pendulum System

Differentiating output y in (5e), the relative degree is obtained by 2 at $x=0$.

Now considering,

$$z=\begin{bmatrix}z_1\\z_2\end{bmatrix}=\begin{bmatrix}h(x)\\L_fh(x)\end{bmatrix}=\begin{bmatrix}x_3\\x_4\end{bmatrix} \quad (6)$$

where, $L_fh(x)=\frac{\partial h(x)}{\partial x}f(x)$ is the lie derivative of $h(x)$ w. r. t. $f(x)$.

The dynamics of IPS (5) can be rewritten in the normal form given as,

$$\dot{z}=\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}L_f h(x)\\ \bar{b}(z)+\bar{a}(z)u\end{bmatrix}\equiv\begin{bmatrix}x_4\\ f_4(x)+g_4(x)u\end{bmatrix} \quad (7)$$

$$y=z_1\equiv x_3$$

where, $\bar{a}(z)=\dfrac{c\cos(x_3)}{ab-c^2\cos^2(x_3)}$;

$$\bar{b}(x)=\frac{c^2\sin(x_3)\cos(x_3)\left(cg\cos(x_3)-ax_4^2\right)}{a\left(ab-c^2\cos^2(x_3)\right)}+\frac{cg\sin(x_3)}{a}.$$

III. Problem Formulations

Consider the inverted pendulum system (7) with uncertain chain of integrators which can be expressed as,

$$\begin{aligned}\dot{z}_1&=z_2\\ \dot{z}_2&=\bar{b}(z)+\bar{a}(z)u+d(t)\\ y&=z_1\end{aligned} \quad (8)$$

where, $d(t)$ is disturbance which is bounded by $\Delta_0>|d(t)|$.

The control objective of this paper is to design a nonlinear robust controller for inverted pendulum system using continuous sliding mode control algorithm, where the actual pendulum angle $y=\theta$ converge to the desired pendulum angle $y_d=\theta_d$ and the tracking error converges to zero, i.e., $\lim_{t\to\infty}(\theta-\theta_d)=0$.

IV. Continuous Sliding Mode Controller Design

In this section, the robust control for inverted pendulum system is designed using continuous sliding mode approach.

Theorem 1: Let k_1 and k_2 are the positive constant such that the polynomial $s^2+k_2s+k_1$ is Hurwitz, and there exists the positive constants k_3 and k_4 with $\varepsilon\in(0,1)$ such that for every $\alpha\in(1-\varepsilon,1)$, the origin is globally stable equilibrium for the system (8) under the feedback controller given by [16],

$$u=\left[\bar{a}(z)\right]^{-1}\left[u_{nom}+u_{stc}-\bar{b}(z)+\ddot{y}_d\right] \quad (9)$$

where, $u_{nom}=-k_1|e_1|^{\alpha_1}\operatorname{sgn}(e_1)-k_2|e_2|^{\alpha_2}\operatorname{sgn}(e_2)$ (10)

where $e_1=z_1-z_{1d}$, with $z_{1d}=y_d$ = desired trajectory, and $e_2=z_2-\dot{y}_d$. Here, α_1 and α_2 are obtained by [16],

$$\alpha_{i-1}=\frac{\alpha_i\alpha_{i+1}}{2\alpha_{i+1}-\alpha_i},\quad \forall i=2 \quad (11)$$

with $\alpha_3=1$ and $\alpha_2=\alpha$, then the value of $\alpha_1=\dfrac{\alpha}{2-\alpha}$.

and $u_{stc}=-k_3|s|^{1/2}\operatorname{sgn}(s)+v$ (12a)

where $\dot{v}=-k_4\operatorname{sgn}(s)$ (12b)

The sliding surface is selected as,

$$s=e_2-e_{20}-\int_0^t u_{nom}\,d\tau \quad (13)$$

Proof: The proof of the above theorem is as follows.

Differentiating (13), becomes

$$\dot{s}=\dot{e}_2-u_{nom}=\dot{z}_2-\ddot{y}_d-u_{nom} \quad (14)$$

$$\dot{s}=\bar{b}(z)+\bar{a}(z)u+d(t)-\ddot{y}_d-u_{nom} \quad (15)$$

Substituting the value of u from (15), (21) becomes

$$\dot{s}=u_{stc}+d(t) \quad (16)$$

Using (12), (16) gives

$$\dot{s}=-k_3|s|^{1/2}\,sign(s)+v+d(t) \quad (17a)$$

$$\dot{v}=-k_4 sign(s) \quad (17b)$$

Now, let us suppose that $\hat{z}=v+d$ and d is Lipschitz disturbance. Therefore,

$$\dot{s}=-k_3|s|^{\frac{1}{2}}\operatorname{sgn}(s)+\hat{z} \quad (18a)$$

$$\dot{\hat{z}}=\dot{v}+\dot{d} \quad (18b)$$

The controller gains are selected as, $k_3=1.5\sqrt{d_0}$ and $k_4=1.1d_0$. Thus, it can be say that $s=\hat{z}=0$ in finite time. In spite of the disturbance $d(t)$, the sliding surface and its derivation $s=\dot{s}=0$ in finite time. Therefore, (16) gives

$$u_{stc}=-d \quad (19)$$

Thus the expression of u_{stc} is coming with negative sign of disturbance which implies that the disturbance is cancelled out. Closed loop system is governed by the nominal control when the system will be on sliding surface and it is stable by design.

V. Simulation Results

In this section, simulation results are presented to assess the performance of the designed control schemes as discussed in previous section. The simulation is performed using physical parameter provided in Table1.

The desired angle of pendulum for both the controllers is chosen as,

$$y_d=0.3\sin(0.2t)+0.2\sin(0.3t)$$

The initial conditions of the IPS in both controllers are selected as,

$$\begin{bmatrix}0 & 0 & 0.1 & 0\end{bmatrix}$$

The IPS with continuous SMC is operated in the closed loop using the following parameters:

$$k_1=k_2=k_3=k_4=3,\ \alpha_1=\frac{1}{3},\ \alpha_2=\frac{1}{2}$$

The simulation studies of nonlinear robust controllers are done by adding a disturbance term in the plant given by, $d(t)=0.02\sin(t)$.

V.A. Continuous sliding mode control with disturbances:

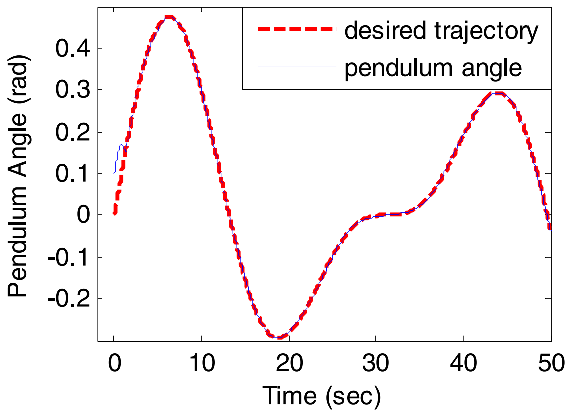

Fig.2 Pendulum Angle with disturbance

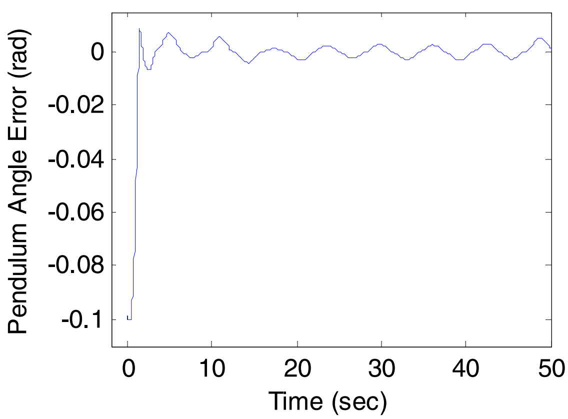

Fig.3 Pendulum Angle error with disturbance

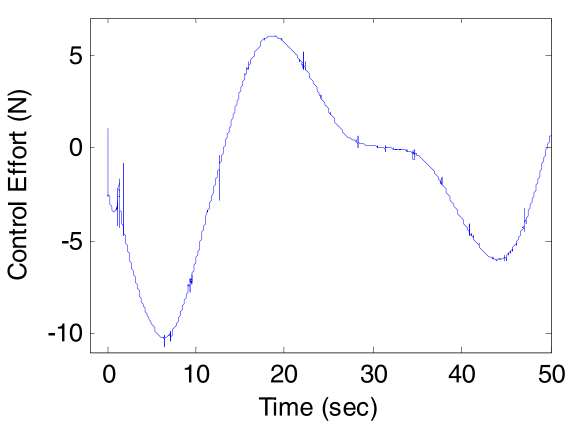

Fig.4 Control effort with disturbance

V.B. Continuous sliding mode control without disturbances:

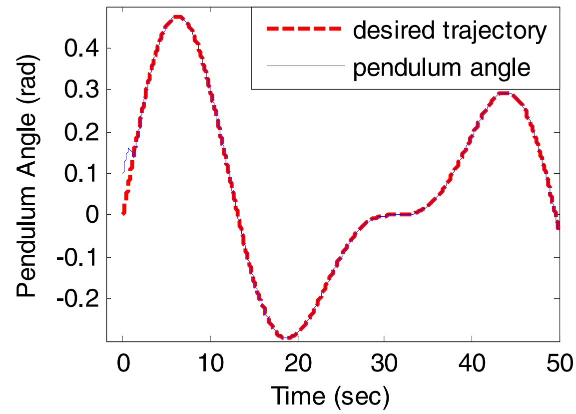

Fig.5 Pendulum Angle without disturbance

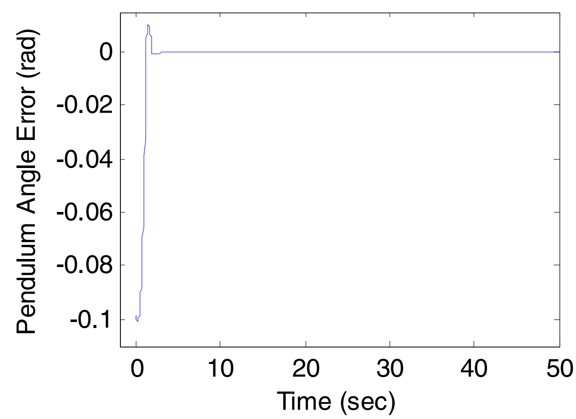

Fig.6 Pendulum Angle error without disturbance

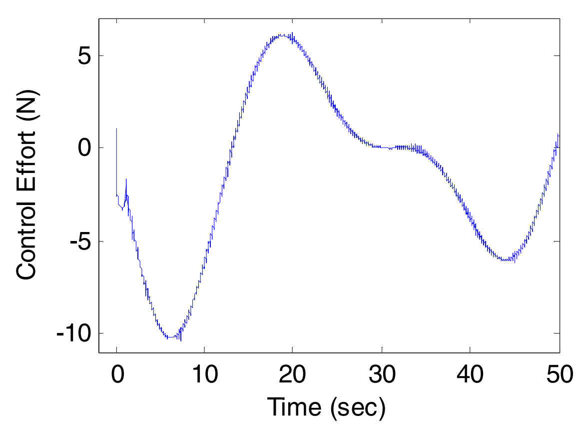

Fig.7 Control effort without disturbance

The reference tracking of actual with desired pendulum angle of IPS with and without disturbance are given in Fig.2 and Fig.5 respectively. The pendulum angle tracking error with and without disturbances are shown in Fig.3 and Fig.6 respectively. The bounded control effort applied to the cart of the inverted pendulum with and without disturbances is illustrated in Fig.4 and Fig.7 respectively.

V.C. Comparative Analysis:

On comparing Fig.3 and Fig.6 the following observation can be made. The reference tracking error in both the cases are converging to zero very quickly. The root mean square (RMS) values of the tracking errors are presented in the Table II.

Table II: RMS value of the tracking error with and without disturbances

Specification	With Disturbances	Without Disturbances
Pendulum angle tracking error	0.014019	0.004179

VI. Conclusions

A nonlinear continuous sliding mode based robust controller for the inverted pendulum is presented in this paper. CSMC approach is applied to cancel out the discontinuous terms present in the ISMC based controllers. Thus, the chattering effect has been discarded completely from the system. The simulation results reveal that proposed control scheme showing satisfactory performance in the presence of disturbances. The adaptive control law with proposed robust control scheme has been worked out which may be reported in near future.

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