Design of Modified Matched Wavelet Design Using Lagrange Interpolation

Manish Kumar Saini
Electrical Engineering Department,
DCR University of Science & Technology, Sonipat
E-mail: memanishkumar@gmail.com

Rajendera Kumar Beniwal
Electrical Engineering Department,
DCR University of Science & Technology, Sonipat
E-mail: mail.rajendera@gmail.com

Abstract: **This paper introduces a technique for designing an optimized matched wavelet by fractionally delaying the coefficients of the matched filter (FDMW). The method is an extension to Matched Wavelets Design process where the filter coefficients are statistically matched to the non-stationary discrete data. Filter coefficients obtained for such unit delay matched filters (UDW) have been fractionally delayed. Fractionally delayed filter coefficients are used to design Matched Wavelet followed by computation of the energy compaction ratio. The results show a significant improvement in the compression using FDMW over UDW. Designed FDM wavelets have lower energy compaction ratio and higher compression rate than given Unit Delay Wavelets.**

***Keyword**: Wavelet, Energy compaction ratio, Lagrange multiplier.*

I INTRODUCTION

Filters are essential in the field of signal processing as these can helpful in desired shaping of signal spectrum or to compute convolution, differentiation and integration type mathematical operation. Filters are generally used for two types of applications, i.e., in separation of combined signals and in restoration of distorted signals. Now days, wavelet is very popular technique for non-stationary data in signal processing. For the best representation of a signal, matched wavelets are designed for the specific signal. In the discrete non stationary data analysis, matched filters have their importance. Fractional delay filter is used to delay the coefficients of the filter by a fraction. Fractional delay filter performs interpolation between the samples of a band limited signal. There are a number of applications of fractional delay filters in speech processing, communication systems, array and antenna processing and in music industries.

Lagrange interpolation, a commonly used tool in the field of signal processing, is mainly used for the interpolation of band limited signals in sampling rate conversion and in fractional delay filters. Lagrange interpolation is used to create the fractional delay in finite impulse response filters to have the coefficients of fractional delay filter. There are many different ways to calculate the coefficients of a Lagrange FD filter. In case of ideal fractional delay filter, maximally-flat approximation is proposed by authors and at any point on which approximation error and its N derivatives are zero, is chosen as zero frequency and then solution is equivalent to Lagrange interpolation [1].

In [2], authors have proposed a technique to find out coefficients of even order Lagrange fractional delay filter using binomial window from truncated sinc function. Here, authors show that the window method, Lagrange interpolation and maximally flat error method are equivalent when expansion point in maximally flat error method is considered as zero angular frequency. Valimaki shows that it also holds for odd order Lagrange fractional delay filter [3]. Sinc interpolation is converged by Lagrange interpolation as the order of the filter approaches infinity [4]. In [5], authors show that the truncated Lagrange FD filter provides a wider bandwidth than Lagrange filter.

In this paper, a new matched wavelet design technique is proposed. In the proposed technique, optimum coefficients are used and the coefficients of matched filter are delayed by a fraction d, which lies between 0 and 1. The basic concept is an extension to matched wavelet design technique which statistically matches filter coefficients with non-stationary discrete data. By using Lagrange multiplier, the coefficients of the unit delay matched filter are fractionally delayed and these delayed coefficients are used for designing the fractionally matched wavelet. Corresponding to coefficients of each delay, energy compaction ratio has been calculated and the coefficients with lowest ECR have been selected as optimal. These optimized fractional delay matched coefficients are named as fraclet, which are used to produce high compression rate and low ECR valued matched wavelet.

In this paper, Section 2 introduces the basics of wavelet transform. Section 3 presents Lagrange interpolation and design of fractional delay filter is in section 4. Performance evaluation of the proposed modified wavelet is presented in section 5. Section 6 concludes the paper and suggests future work for the reader.

II WAVELET TRANSFORM

Wavelet Transform is most commonly used method in signal processing applications. It is very useful in non-stationary signal analysis. Wavelet transform has the capability to give concurrent information about time and frequency of a signal.

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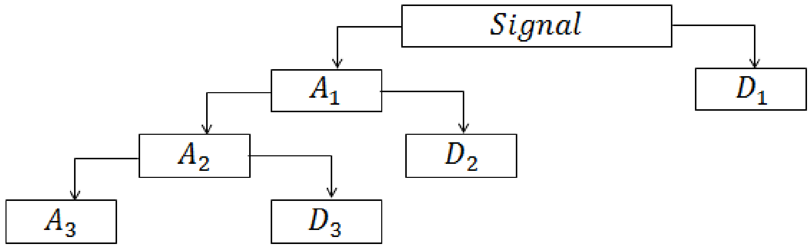

Fig.1 Wavelet decomposition

To separate frequency component of different frequencies, it is very useful technique. WT can be classified as continuous wavelet transforms (CWT) and discrete wavelet transforms (DWT). WT has two main parameters i.e., scaling and translation which are associated with mother wavelet function. Performance of WT is based on choice of mother wavelet for particular application [7].

WT uses the multi-resolution analysis (MRA) technique, which divides the input signal into low frequency and high frequency components. Low frequency components give the approximation coefficients (A_j). High frequency components give the detailed coefficients (D_j) [8].

The wavelet function is given as

$$\psi(k) = \sqrt{2}\sum_n h_p(n)\,\phi(2k-n) \qquad (1)$$

Scaling function

$$\phi(k) = \sqrt{2}\sum_n l_p(n)\,\phi(2k-n) \qquad (2)$$

where n= no. of samples (1, 2……M)

III LAGRANGE MULTIPLIER

Lagrange interpolation is frequently used in signal processing. It is used for the interpolation of band limited signals, for instance, in sampling rate conversion and in fractional delay filter. There are many approaches for designing a FD FIR filter [9, 10]. Lagrangian interpolation method has been selected to get the maximally flat magnitude response. Maximally flat FIR approximation is used for obtaining the Lagrange interpolation. Different methods to obtain it have been given in [1]. The following equation gives the coefficients of Lagrange interpolator,

$$C_L(n) = \prod_{\substack{k=0 \\ k\neq n}}^{M} \frac{(d-k)}{(n-k)} \quad \text{for } n = 0,1,2,\ldots,M \qquad (3)$$

where d is a real number that corresponds to the delay and M is the order of Lagrange FD filter.

The above equation can be obtained by windowed method [2], with the condition that window function must be scaled to binomial window. The equation of windowing method is given below for both even and odd N,

$$h(n) = C_{bin}(d,M)w_{bin}(n)\mathrm{sinc}(n-d) \qquad (4)$$

where $n = 0,1,2,\ldots,M$

Scaling coefficient is defined as

$$C_{bin}(d,M) = (-1)^M \frac{\pi(M+1)}{\sin(\pi d)} \binom{d}{M+1} \qquad (5)$$

Binomial window is defined as

$$w_{bin}(n) = \binom{M}{n} \qquad (6)$$

IV. Design of Fractional Delay Filter

In this section, the basics of design process have been presented. Simply to design FD filter, the coefficients of FIR filter are delayed by fraction and after obtaining these coefficients, matched wavelet is designed.

The optimal information is retrieved using Lagrange interpolation. The method takes existing unit delay (UD) Filter coefficients and convolves them with fractional delay filter coefficients. This convolution produces fractionally delayed coefficients. The closed-form formula to compute the coefficients of an M^{th}order Lagrange FD filter, $C_L(n)$is given as

$$C_L(n) = \prod_{\substack{k=0 \\ k\neq n}}^{M} \frac{(d-k)}{(n-k)} \qquad (3)$$

for $n = 0,1,2,\ldots,M$

M is Lagrange FD filter order and d is delay.

Let $C_U(n)$be the coefficients of the existing unit delay filter. The FDM filter coefficients $C_F(n)$ are given by

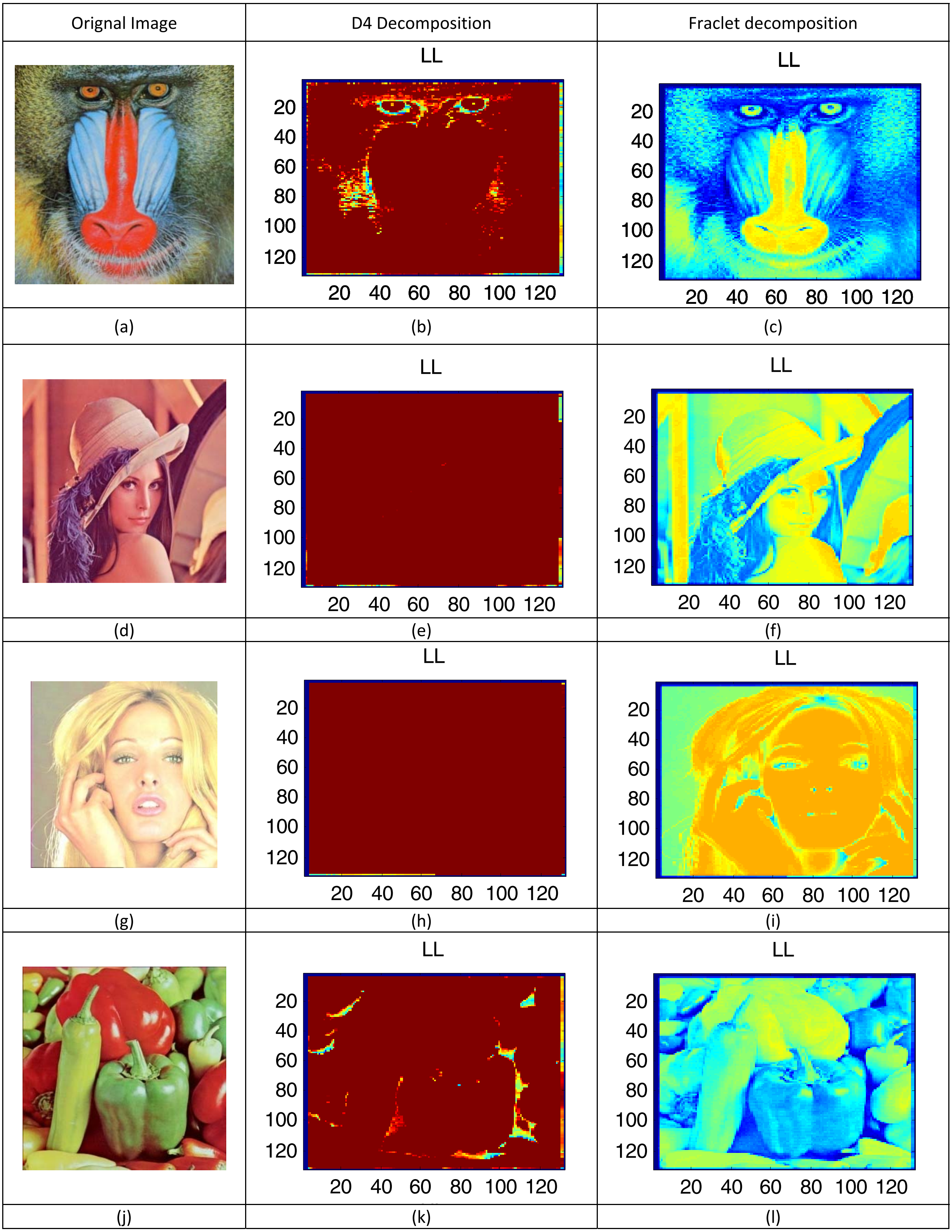

Figure 2. Decomposition levels for FDMF and UD Filter

Table 1: Effect of FD on ECR and FD filter coefficients

S. No	FD, d	ECR	Filter Coefficients (D4)
1.	0.1	*0.3311*	[0.5645 0.3259 -0.0414 -0.0052]
2.	0.2	0.4455	[0.4590 0.5962 -0.0710 -0.0088]
3.	0.3	0.5582	[0.3658 0.8145 -0.0899 -0.0109]
4.	0.4	0.6560	[0.2841 0.9843 -0.0989 -0.0117]
5.	0.5	0.7307	[0.2134 1.109 -0.0991 -0.0114]
6.	0.6	0.7778	[0.1530 1.1925 -0.0913 -0.0102]
7.	0.7	0.7962	[0.1021 1.2380 -0.0766 -0.0083]
8.	0.8	0.7869	[0.0601 1.2493 -0.0558 -0.0059]
9.	0.9	0.7529	[0.0263 1.2297 -0.0300 -0.0030]
10.	UD	0.7529	[0.6830 1.1830 0.3169 -0.1830]

$$C_F(n) = \left[k \prod_{\substack{k=0 \\ k\neq n}}^{M} \frac{(d-k)}{(n-k)}\right] . C_u(n) = C_L(n). C_U(n) \qquad (7)$$

For comparison purpose, we have taken D4 which is the most compact filter.

V. Performance Evaluations

In this section, performance of the proposed technique is analyzed by taking the four standard test images. In unit delay D4 filters and unit delay filter used in Gupta et al., coefficients of D4 filter are selected because this is the best in Daubechies family. The filter design in ref. [6], is the suitable for non-stationary discrete data compression such as image and speech applications. For evaluating the performance of the fractional delay filter, Energy compaction ratio (ECR) is selected as an important decomposition. ECR has its lower value at $d = 0.1$ for FDM D4 filter and at $d = 0.9$ for FDM filter as in Table 1. Optimized FDM filter coefficients are based on lower ECR which increases the decomposition levels.

Figure 2 shows the decomposition of the standard images. In this figure, first column shows the original images and second column presents the images after being decomposed by Unit Delay D4 filter coefficients at first level. Third column has the decomposed images after being treated with the proposed FDM filter coefficients.

In the Figure 2, it can be analyzed that the image obtained after FDM decomposition is easily recognizable. Where the image obtained after UD D4 decomposition seems of no use. Here, the image obtained after FDM decomposition can be further decomposed to get more detailed information. The images have been treated with the Unit Delay filter having the coefficients given in equation (2). The results of the decomposition of these images have been provided in Figure 2. Even at the first level of decomposition, the resultant image is not recognizable in the case of UD D4 filter whereas it is clearly recognized in the case of FDM D4 filter (Figure 2). Same thing occurs at the second decomposition level. As we go to the third decomposition level, the FDM D4 filtered image still has some information against no information present at third decomposition level of UD D4 filter (Figure 2). The presence of useful information at the third level of decomposition in the case of FDM D4 filter showcases the outperformance of FDM D4 filter [6] over the UD D4 filter. Table 1 shows the coefficients of D4 filter with the different values of d ($0<d<1$). Correspondingly the energy compaction ratio also has been calculated for different values of d. These results show that the proposed design of matched wavelets performs better after the coefficients of matched filter are fractionally delayed.

VI. Conclusions

In this work, the fractionally delayed filter is applied in analysis of image pattern. The proposed FDM technique in this paper is an extension of the matched wavelet design process. The matched wavelet filter coefficients have been fractionally delayed to improve the given matched wavelet compression capability. The main advantage of FDM wavelet is that it produces lower energy compaction ratio and higher compression rate. It is inferred from the above analytical results that Fraclet outperforms the unit delay filters.

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