Formulation and Computation of Cattle Feed Mix by Using TORA and LINGO: Minimization of Adverse Effect of Nutrient Ingredient

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***Abstract*— This paper presents mathematical models for multi-criteria objectives for animal diet formulation using LINGO and TORA. It represents formulation and computation techniques with the help of these software. Multi-objectives of this paper are; minimization of cost for cattle diet, better shelf life for feed mix and minimization of adverse effect of nutrient ingredients crude protein and phosphorus. Minimization of adverse effect of nutrient ingredients is done by using goal programming which gives a new dimension to this work.**

***Keywords*— Diet Formulation, goal programming, linear and non-linear programming, adverse effect of nutrient ingredients.**

I. INTRODUCTION

In dairy farming, producers have a variety of objectives. Some of the primary objectives are maximization of weight gain, milk yield and nutrient utilization. Animal diet formulation involves the allocation of feed ingredients with satisfaction of minimum nutrient requirements to achieve specific objectives. A nutritionally balanced ration plays an important role in achieving these objectives. The objective of ration formulation is to allocate a balanced ration to animal which fulfill their nutrient requirement at different stages of production and livestock. These animal diet formulation models are used for commercial purposes as well as for livestock development, and have been developed using different forms of mathematical programming for last many decades.

Linear programming technique has been used to define the feeding problem in mathematical form[1]. A review was presented based on programming techniques used for animal diet formulation chronologically[2]. A linear programming model has been developed to investigate, analyse and determine the most efficient way of finding the least cost ration for broilers of age 6 to 10 weeks for the utilization of locally available and non- conventional feed stuff-Duckweed (Lemna paucicostata) [3]. A mathematical model has been presented to identify optimal beef production systems in Ireland. The objective function is to, maximize farm gross margin with a constraint set of animal nutritional requirements[4]. Nonlinear programming techniques were used for weight gain in sheep [5]. A model was developed by Tozer to achieve a final calving weight of 600 kg for large-breed replacement dairy heifers and then variability of crude protein content of ration ingredients has been incorporated by three methods; right-hand side adjustment, incorporation of a safety margin and stochastic programming[6]. A model was developed to minimize the risk of not meeting the nutrient requirements through under formulation and over formulation which will result in high cost[7]. The nutritional balance of the raw materials selected by GP showed a marked improvement over those of LP[8]. A model was developed by combination of linear programming and weighted goal programming[9]. A goal programming model has been developed in which the goals of meal quality have been introduced and different requirements of decision makers have been modeled[10]. A tool based on a three phase optimization approach has been presented for pigs' daily ration formulation which enables decision makers to find the optimal energy content of the pigs' diets, which changes frequently due to rapidly fluctuated economic circumstances[11]. Linear and nonlinear programming techniques were compared for animal diet formulation[12]. The nutritional requirements of dairy cattle are different at different weights[13].

In this paper, the research objectives are taken as cost minimization of ration and maximization of nutritional value of the ration at different weight stages. To achieve the optimal feed ration, cost of the feed ingredients, nutrient requirement of dairy cows have been considered as input data. Objectives of this paper are as follows:

- The first objective in this paper is to get a roughly balanced feed ration at least cost for different weight stages. For these objective, feed components that contain high digestibility ingredients have been chosen.
- Second objective is to formulate a ration which maximizes the quality of feed ration in terms of its shelf life, which can be increased by reducing the water content of feed ration. Reduced water content allows reduced feed costs at same weight gain with a smaller feed ration quantity.
- Third objective is to minimize the sum of the over-achievement of the nutrient ingredients.

Standard Linear programming has been used to achieve these objectives for different stages of livestock. Stochastic programming is used to introduce variability in nutrient ingredient and to overcome the deterministic assumption of linear programming. Multi objective Goal programming technique has been combined with linear programming to get more reliable results from feed and nutrition point of view. It is also used to minimize the adverse effect of nutrient ingredients.

II. Material and Methods

In these models 16 feed ingredients and 5 nutrients have been used for optimization of feed blend.

Input data for determination of feed blend

Following notations have been used :

z objective function, c_j per unit cost of feed ingredient j , x_j quantity of jth feed ingredient in the feed ration, a_{ij} amount of nutrient i available in the feed ingredient j , b_i minimum requirement of ith nutrient , i index identifying feed nutrient components with i = 1,2,….m , j index identifying feed components with j =1,2,………n.

Input data for cost, water content and nutritional composition of feed ingredients is shown in table I.

Table II represents minimum requirement for different nutrients at different weight of dairy cattle.

Table II
Minimum Requirement for different nutrients at different weight of dairy cow to reach at 600 kg weight

Nutrient	200 kg	300 kg	450 kg	600 kg
ME(MJ)	43.710	57.11	67.37	82.06
CP(g)	533	671	749	879
DM(g)	5000	6670	7870	9580
Ca(g)	18	20	23	25
P(g)	12	15	18	18

Linear Programming Model for determination of feed blend for cost and water content minimization

$$min\ \ z = \sum c_j x_j$$

$$s.t.\ \sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

$$x_j \geq 0, b_i \geq 0$$

Linear programming model has been formulated for obtaining the optimum values of feed ingredients at different stages of livestock. This model is divided into four sub models to:

i. minimize the per kg cost of ration of dairy cow of weight 200 kg
ii. minimize the per kg cost of ration of dairy cow of weight 300 kg.
iii. minimize the per kg cost of ration of dairy cow of weight 450 kg
iv. minimize the per kg cost of ration of dairy cow of weight 600 kg.

Linear programming model has also been formulated to maximize the quality of feed ration in terms of its shelf life. It has been done by minimizing the water content of the feed blend. This model has been divided into four sub models to:

v. minimize the water content of ration of dairy cow of weight 200 kg.
vi. minimize the water content of ration of dairy cow of weight 300 kg.
vii. minimize the water content of ration of dairy cow of weight 450kg.
viii. minimize the water content of ration of dairy cow of weight 600 kg.

These models have been formulated to find optimal value of feed ingredients at different stages of livestock. Now the stochastic programming models will be developed to consider the variability of nutritional values of feed ingredients.

Stochastic Programming Model for determination of feed blend for cost and water content minimization

$$min\ \ z = \sum c_j x_j$$

$$s.t.\ \sum_{j=1}^{n}\left(a_{ij} - z\left(\sqrt{\sum_{j=1}^{n} \sigma_{ij}^{\,2}}\right)\right)x_j \geq b_i$$

$$x_j \geq 0, b_i \geq 0$$

where $\left(\sigma_{ij}^{\,2}\right)$ represents variance of nutrient i in ingredient j, rest of the variables have been defined as above . This model has been formulated assuming probability of 80% which takes the variability of nutritional values of feed component into account through the constraints and term b_i, rest is similar as in LP model. The requested probability determines the nutrient concentration for ration formulation. Nutrient variability has been considered as a nonlinear term of variance in each feed component and a desired probability level. Again eight models eight models have been formulated for sixteen feed ingredients to obtain optimum value of variables at specific levels of livestock. Water content has again been minimized to increase the shelf life of ration.

Four goal programming models for 4 different weight classes of 200kg, 300kg, 450kg and 600kg respectively have been defined. The objective functions of linear programming models have been taken as constraints with deviation variables. Rest of the constraints are same as in linear programming model. Two goals have been formed, one for cost minimization and other for water content minimization. The objective is to minimize the sum of the overachievement of the goals, Rest of the variables have been defined as above.

Goal Programming Model for determination of feed blend

$$\min z = d_1^+ + d_2^+$$

$$s.t. \sum_{i=1}^{16} c_{ik} x_i + d_k^- - d_k^+ = z_k$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

$$x_j \geq 0, b_i \geq 0$$

where k = 1,2.

Results obtained by goal programming models show that the protein content and phosphorous present in feed ration has been exceeded significantly compared to the minimum requirement of these nutrients. Therefore goal programming models have been reformulated with the conversion of the constraints. Corresponding deviational variable for over-achievement are incorporated into objective function.

Goal programming model is introduced to minimize the adverse effect of crude protein and phosphorus. Excess dietary crude protein is converted to ammonia and is toxic to animal. It also makes negative effect of the fertility cycle of animal. It uses extra energy of animal to convert ammonia to urea which limits the amount of fat available for milk production. Overdose of phosphorus also effects reproductivity of animal. Goal programming technique is used to minimize the over achievement of these two nutrient ingredients.

Goal Programming model to minimize the over-achievement of nutrient ingredients

$$\min z = d_1^+ + d_2^+ + d_3^+ + d_4^+$$

$$s.t. \sum_{i=1}^{16} c_{ik} x_i + d_k^- - d_k^+ = z_k$$

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

$$x_j \geq 0, b_i \geq 0$$

k=1, 2, 3, 4

III. Discussion

All diet formulation system modeled by linear programming is solved by TORA. Optimum values of feed ingredients for minimum cost and for minimum water content to reach different weight class of dairy cattle are obtained by these model solutions. Stochastic programming models also provides the minimum cost and minimum water content but the difference between two results are that stochastic programming results includes variability of nutrient content. That goal of adding nutrition variability to animal diet is achieved by introducing the variance term in the stochastic programming models. Goal programming models minimize the sum of deviation of decision variables for both of the above said objectives. LINGO is used to solve goal programming models. 24 models are presented and analyzed by the blend of linear programming, stochastic programming and goal programming. Table III represents minimum cost and minimum water content for different level of livestock by using the blend of the three techniques linear programming, stochastic programming and goal programming. Table IV provides solution values of feed ingredients for cost minimization by linear and stochastic programming.

Table III

Minimum cost for different weight gain range.

For minimum cost			For minimum water content		
Weight	Linear Programming	Stochastic Programming	Weight	LP	SP
200	51.29	69.87	200	.52	.56
300	66.35	89.89	300	.69	.74
450	76.87	105.80	450	.81	.87
600	92.34	126.41	600	.99	1.05

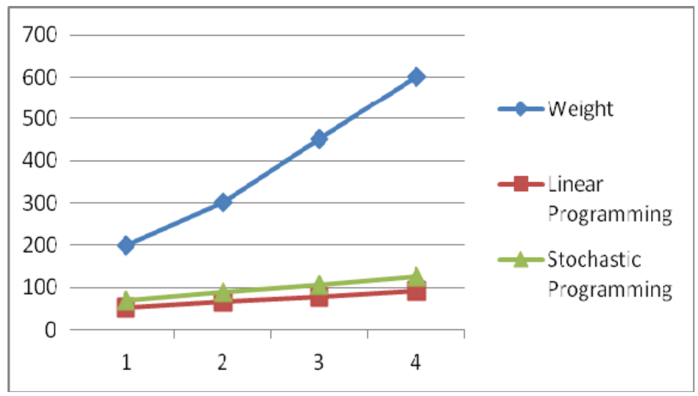

Fig.1 Representation of minimum cost by linear and stochastic programming

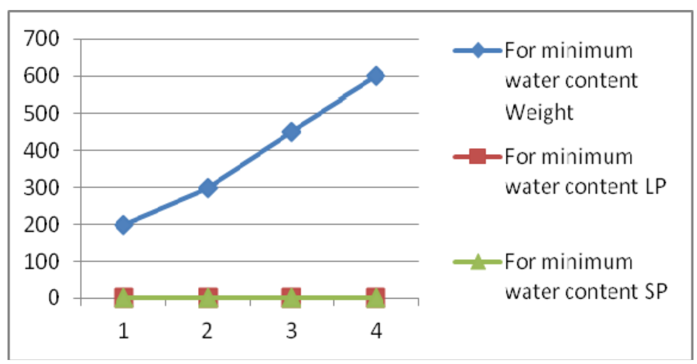

Fig.2 Representation of minimum water content by linear and stochastic programming

Fig. 1 and fig. 2 provides graphical representation of minimum cost and minimum water content by linear and stochastic programming respectively.

Table VI

Solution value of feed ingredients for feed ingredients by goal programming

		Weight Gain Range			
S. No.	Feed Ingredients	200	300	450	600
x_1	Alfalfa hay	0.54	0.49	0.54	0.67E-02
x_2	Barley grain	0	0.47E-02	0	0.89E-01
x_3	Sugarbeetpulp	2.49	3.13	3.69	4.83
x_{11}	Rice bran(fibre 11-20%)	2.30	3.01	3.59	4.1
x_{13}	Wheat straw	0.28	0.83	1	1.70
x_{15}	Canola meal(solvent-extracted)	0.43E-03	0	0	0

Results obtained for cost minimization shows that when cost is optimized by stochastic programming technique, it is providing the better results in the sense of nutritional variability as compared to linear programming technique. It also includes variability of nutrient components at slightly higher cost. Graphical representation of comparison of results by linear and stochastic programming is shown by fig. 1. Table III represents the total costs for a feed blend to reach dairy cattle through each weight class at specific growth rate. It provides the results for zero deviations of above defined LP and SP models.

Table VII

Deviation variable values by goal programming

Deviation variable	Weight Gain Range			
	200	300	450	600
D_1	0	0	0	0
D_2	0	0.42E-02	0.23E-02	0.39E-02
D_3	0	0	0	0
D_4	0.28E-01	0.36E-01	0.44E-01	0.57E-01

Table VII provides deviation variable values by goal programming. D_1 and D_2 represent under-achievement and over-achievement of cost respectively, while D_3 and D_4 represents under-achievement and over-achievement of water content respectively It shows that most of the weight range has zero deviations except for D_4. It is clear from the results shown in table 4, that more number of variables is included in the diet, if modeled by stochastic programming technique. It should also be pointed out that marginal values of variables are also greater in this case. Results, obtained from goal programming helps to minimize the deviations in the results. These results provide the value of feed components obtained for different weight classes. Linear programming models and stochastic models are used with the constraints such that all nutrients at least achieve the NRC requirements.

Table VIII

Deviation variable values by goal programming with Crude protein and Phosphorus as goals

Deviation variable	Weight Gain Range			
	200	300	450	600
D4	0.22E-01	0.30E-01	0.38E-01	0.57E-01
D7	0	0	0	0.49E-03

Table VIII provides the value of deviations for D_4 and D_7, which represents deviation for over-achievement of water content and under achievement of phosphorus.

IV. CONCLUSIONS

This paper provides a combination of mathematical programming (Linear, Stochastic and Goal programming) to optimize the livestock feed mix. The paper represents a bi-criteria model and successful application of SP and goal programming models. Models presented in this paper minimize the cost and provides the feed mix with better shelf quality. It also represents the mathematical model to minimize the adverse effect of phosphorus and crude protein the diet and it makes the work different from the previous studies. Future work is proposed as minimization of adverse effect of more variables simultaneously.

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Table 1 Composition of feed ingredients with cost, water content and nutritional composition

Table I

Composition of feed ingredients with cost, water content and nutritional composition

Notation	Feed ingredients	Price	Water content	Metabolizable energy(ME)(mj/kg of feed)	Crude Protein (CP)(g/kg)	NDF (g/kg)	DM (g/kg)	Ca (g/kg)	P (g/kg)
x_1	Alfalfa hay	14	.11	7.51	163	400	894	15	2.3
x_2	Barley grain	10	.13	10.80	103	189	871	0.7	3.4
x_3	Sugarbeetpulp	15	.11	9.99	83	429	892	13.83	0.89
x_4	Cottonseed meal (high fibre, low oil)	18	.10	9.2	360	330	902	2.62	11
x_5	Soyabean meal(high protein-dehulled)	28	.12	11.98	471	97	881	3.17	6.70
x_6	Sunflower meal(solvent-extracted, dehulled or non-dehulled)	16	.11	8.10	288	400	890	3.92	10.32
x_7	Wheat bran	19	.13	9.57	151	394	870	1.22	9.66
x_8	Maize grain high moisture	23	.35	8.84	62	89	650	0.32	2.01
x_9	Sorghum grain	17	.13	11.80	94	96	874	0.26	2.88
x_{10}	Groundnut meal(solvent-extracted)	25	.11	11.16	489	217	893	1.52	5.54
x_{11}	Rice bran(fibre 11-20%)	10	.10	9.11	115	310	902	0.63	12.45
x_{12}	Oats grain	18	.12	8.70	97	314	879	0.97	3.16
x_{13}	Wheat straw	7	.09	6.19	38	706	910	4.37	0.64
x_{14}	Corn gluten feed	14	.12	10.77	192	350	883	1.41	9.01
x_{15}	Canola meal(solvent-extracted)	24	.10	10.54	351	242	901	6.67	10.45
x_{16}	Cottonseed hulls	11	.10	5.89	46	773	906	1.18	0.91

Table IV

Solution value of feed ingredients for cost minimization by linear and stochastic programming

	Feed Ingredients	Linear Programming				Stochastic Programming			
		200	300	450	600	200	300	450	600
x_1	Alfalfa hay	0.05	0	0	0	0.54	0.48	0.54	0
x_2	Barley grain	1.56	.59	.61	0	0	0	0	0.08
x_3	Sugarbeetpulp	0	0	0	0	2.49	3.14	3.69	4.84
x_4	Cottonseed meal (high fibre, low oil)	.66	.5	.4	.16	0	0	0	0
x_{11}	Rice bran(fibre 11-20%)	0	2.46	3.23	5.52	2.3	3.02	3.59	4.10
x_{12}	Oats grain	0	0	0	0	0	0	0	0
x_{13}	Wheat straw	3.30	3.83	4.46	4.9	0.28	0.83	1	1.71

Table V

Solution value of feed ingredients for water content minimization by linear and stochastic programming

	Feed Ingredients	Linear Programming				Stochastic Programming			
		200	300	450	600	200	300	450	600
x_1	Alfalfa hay	0	0	0	0	0.34	0.22	0.23	0
x_2	Barley grain	0	0	0	0	0	0	0	0
x_3	Sugarbeetpulp	0	0	0	0	1.12	1.34	1.55	1.85
x_{12}	Oats grain	0	0	0	0	0	0	0	0
x_{13}	Wheat straw	3.32	4.69	5.54	6.74	1.3	2.17	2.59	3.33
x_{14}	Corn gluten feed	0	0	0	0	0	0	0	0
x_{15}	Canola meal(solvent-extracted)	2.2	2.66	3.14	3.83	2.83	3.71	4.42	5.51