Transmission Loss Allocation using Fairness index in Deregulated Market

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Abstract: Deregulated market provides opportunities to consumer and generator for transparent cost determination. Generator side revolution invites a lot of thoughts and ideas to make electricity market more competetitive. For sustainable market operation, each component issues should be taken care. Transmission line is vital part of deregulated market. Therefore, losses occurring in transmission line should be considered. Cross-term allocation of transmission losses has been done with various ways but none were satisfactory and acceptable to all. Transmission loss allocation to various loads connected from bus or node becomes complex owing to nature of quadratic expression. Quadratic expression of line loss makes it almost impossible to separate loss for various users. This paper presents a new method for transmission loss allocation with consideration of fairness factor. This new method allocates transmission losses in such a way that distribution of a single load in to further various loads does not change overall percentage loss allocation.

Proposed method has been tested on IEEE-57 bus system to test result. The proposed method is simple and time efficient.

Keywords: Fairness criterion, Gutenburg-Richter's law Allocation, loss allocation, Power law Allocation

I. Introduction

Electric industry witnesses huge change with more competition in style of operation, generation, transmission and distribution. Each part has its own significance from the point of view of reliable supply to consumers. Generator cost parameter optimization has travelled miles for improvement and many significant results have been accepted. In last three decades of restructuring of power system was mainly focused on generator side. But apart from generation, electricity transmission is integral part of it. Transmission lines are not only used for transmission but it is also used for ancillary services like VAR/ voltage control, load-frequency control, metering and billing.

Therefore, transmission line losses should be properly recovered for sustainable operation of power system. Owing to non-linear characteristics of line losses, it becomes very difficult to fairly allocate and recover losses from parties participating in deregulated market. Method adopted for transmission loss allocation should be fair nough so that it can be accepted by all. This is only possible when adopted method is transparent and fair.

Cross-term loss allocation of transmission losses has been well researched. Pro-rata (PR), marginal procedure and proportional sharing procedures are different practical algorithm which created impact in transmission loss allocation [1]. A PR method of loss allocation has been adopted by deregulated market of mainland Spain where complete transmission losses are allocated to consumers and proportional loss allocation method is applied to allocated percentage loss allocation among various loads. But proportional loss allocation method is not fair as on the scale of fairness index.

Marginal procedure method adopted in Norwegian deregulated electricity market. In this method losses are allocated to generators and loads (consumers) on the basis of incremental transmission losses (ITL). Paper [2] and [3] gives detailed discussion for ITL procedure.

Proportional sharing procedure allocates transmission losses to generators and consumers proportional to magnitude of active power contribution in network. This method assumes that power reaching to any node or bus through various lines shares losses proportional to magnitude of power flow in lines. But losses in lines are quadratic proportional to magnitude of current. Therefore, this procedure cannot be considered fair. References [5] address the issue of fair loss allocation of cross-term with proportional allocation, quadratic allocation and geometric allocation. In these methods of loss allocation cross-term has been distributed based on magnitude, square of magnitude and logarithmic of magnitude. These method are accepted but it did not tell us which one is fairer than others.

Reference [6]-[7] followed tracing of power flow in various lines and asses the loss contribution. But it is quite complex and cumbersome to trace the power flow in larger network. Game theory, artificial neural network and circuit theory based methods have also been implemented to allocate transmission losses. Results with game theory were optimistic but did not address the issue of cross-term allocation. To measure fairness of any method has not been discussed so far.

This paper presents a new approach of transmission loss allocation with concept of fairness index. Fairness index is used to measure and validate any given algorithm. Author proposes loss allocation with Gutenburg-Richter's allocation (G-RA).

II. Loss Allocation Methods

Cross-term transmission loss allocation for various load supplied from given bus or node discussed and proposed with the help of artificial neural network [1]-[10] game theory[11]-[14] and others methods. Power supplied to any load gets power through various lines connected to load. Therefore, each line is carrying and contributing supply to load. Now total power received by node is causing transmission loss. But which load is receiving how much from any line is issue to be discussed. If transmission line has flow of $(P_i + P_j)$ to supply load of P_i and P_j at load as shown in figure-A.

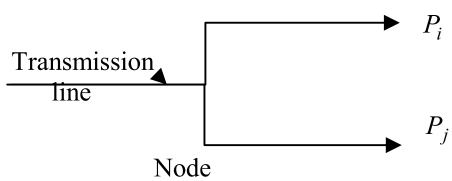

Figure-A: Load distribution at Node

As in the figure, transmission line carries power ($P_i + P_j$) which gets distributed in two loads P_i and P_j. In such case, transmission loss occurring in line will be $(P_i + P_j)^2 * r = P_i^2 * r + P_j^2 * r + 2 * P_i * P_j * r -----(1)$

Where, r is the resistance of transmission line. The loss term $P_i^2 * r$ is agreed upon by P_i and $P_j^2 * r$ will be agreed upon by P_j. But cross-term ($2 * P_i * P_j * r$) allocation is the issue to distribute between P_i and P_j.

If losses are distributed as part β_i and β_j between P_i and P_j then,

$$\beta_i + \beta_j = 2 -----(2)$$

In proportional allocation part of allocated loss will be directly proportional to magnitude while quadratic allocation allocates according to square of magnitude and geometric shares according to logarithmic of magnitude. If β_i and β_j could be find out then Transmission loss allocated to load P_i can be given by,

$$= P_i^2 * r + \beta_i * P_i * P_j * r -----(3)$$

Similarly, Transmission loss allocated to P_j,

$$= P_j^2 * r + \beta_j * P_i * P_j * r -----(4)$$

To find β_i and β_j, different allocation schemes like proportional allocation, quadratic allocation and geometric allocation have been proposed by[5]. The paper presents a new scheme for allocation with fairness index allocation. Each of method of paper [5] has been discussed and mathematical steps are given in detail as follows,

a. Proportional Allocation(PA)

$$\frac{\beta_i}{\beta_j} = \frac{P_i}{P_j} -----(5)$$

$$\beta_i = \frac{P_i}{\frac{1}{2}(P_i + P_j)} -----(6)$$

b. Quadratic Allocation(QA)

$$\frac{\beta_i}{\beta_j} = \frac{P_i^2}{P_j^2} -----(7)$$

$$\beta_i = \frac{P_i^2}{\frac{1}{2}(P_i^2 + P_j^2)} \beta_i = \frac{P_i^2}{\frac{1}{2}(P_i^2 + P_j^2)} -----(8)$$

c. Geometric Allocation(GA)

$$\frac{P_i}{P_j} = \frac{e^{\beta_i}}{e^{\beta_j}} -----(9)$$

$$\beta_j = \beta_i - \ln\frac{P_i}{P_j}$$

$$\beta_i = 1 - \frac{1}{2}\ln\frac{P_i}{P_j} = 1 + \log\frac{P_i}{P_g} -----(10)$$

$$P_g = \sqrt{P_i * P_j}$$

These allocation methods were allocating transmission loss among loads at node or bus. But these methods cannot be accepted due to fairness concern. Therefore, to evaluate fairness some criterion should defined which satisfy concern of participants in deregulated electricity market.

III. Fairness Index

Fairness evaluation is well researched issues. Many have been defined fairness on the basis of different concepts and situations [17] and [20]. But one particular method can't be applied all cases. Fairness index defined with proportional fairness and Max-Min fairness.

Fairness index becomes important for any resource allocation process among any number of participants. Each fairness index should possess the feature addressing the concerns of parties involve as well as it should be independent of resource and number of participant. Paper [19] defines such a fairness index to judge the resource allocation methods. This method gives fairness as number between 0 and 1. If a resource is to be allocated among n participants such that the i_{th} user receives an allocation x_i. Fairness index $f(x)$ can be defined as

$$f(x)=\frac{\left[\sum_{i=1}^{n} x_i\right]^2}{n\sum_{i=1}^{n} x_i^{\,2}} -----(11)$$

Fairness does not necessarily mean equal distribution of resources. In some cases it is justified to allocate more resources to particular participants. In such cases fairness index will be the ratio of resources allocated. Fairness index evaluates the method of resource allocation. If fairness index is 0.2 means it is fair only for 20% of the participants while method is unfair for rest of the participants. Therefore, for being a good scheme for transmission loss allocation, allocation method should have fairness index close to unity as possible.

IV. Proposed method

This paper presents two different mathematical model to allocate transmission loss to various load connected to a given node or bus.

a. Power Law Allocation (PwA):

Power law is clearly visible applied in most of the natural phenomenon like population of city, sizes of earthquakes, moon carters, solar flares, frequency of word used in human language, the sales of book, number of hits on web pages and many more [15]. Mathematically, Power law can be expressed as

$$P(x)=Cx^{-\alpha} -----(12)$$

Where, C and α are the constant. Generally α=2.5 for natural phenomenon. Therefore, transmission loss can be allocated among load P_i and P_j according to expression

$$\frac{P_i}{P_j}=\frac{\beta_i^{-\alpha}}{\beta_j^{-\alpha}} -----(13)$$

$$\beta_i=\frac{P_i^{-1/\alpha}}{\frac{1}{2}(P_i^{-1/\alpha}+P_i^{-1/\alpha})} -----(14)$$

This method generalizes the allocation. For different values of α, it defines the proportional allocation and quadratic allocation. Like for α=-1, power law allocation becomes proportional allocation and for α=-2, power law allocation becomes quadratic allocation.

b. Gutenburg-Richter's law (G-RA):

Following Gutenburg-Richter's law [18], Transmission loss can be allocated as

$$\frac{P_i}{P_j}=\frac{e^{-b\beta_i}}{e^{-b\beta_j}} -----(15)$$

$$\beta_j = 1+\frac{1}{2b}\ln\frac{P_i}{P_j} -----(16)$$

$$\beta_i = 1-\frac{1}{b}\log\frac{P_i}{P_g} -----(17)$$

$$P_g=\sqrt{P_i * P_j}$$

P_g is geometric average of P_i and P_j. For b=-1, Gutenburg-Richter's law allocation becomes equivalent to geometric allocation. In summary, Power law allocation and Gutenburg-Richter's law allocation are more general allocation techniques than proposed in paper [5]. To get fairness index close to unity, these constant can be varied for range of values.

V. Simulations and results

Proposed methods of transmission loss allocation have been tested on IEEE-57 bus system. IEEE-57 bus standard data taken. ACOPF is used to find operating parameters like voltage magnitude, phase angle, active power and reactive power of both test cases. ACOPF helps in finding active and reactive power flow through lines connected between the nodes. Transmission loss allocation at a node or bus does not affect the parameters of power system. Proposed method distributes the cross-term loss portion among the loads at node or bus. This distribution is assessed by fairness index, which simply tells a number between 0 and 1. Fairness index 0 shows worst fair method while method with fairness index 1 is best.

Table No I: Loss allocation at bus 13

P(MW)	PA	QA	GA	PwA	G-RA
2	8.85	3.92	20.37	24.47	22.79
3	14.20	8.82	22.47	21.75	20.75
5	26.75	24.51	25.94	22.41	22.32
8	50.21	62.75	50.21	31.37	34.14
$f(x)$	0.711	0.540	0.861	0.9773	0.957

Table no-I depicts loss allocation to various load at bus no 13 and figure-I shows corresponding plot of transmission losses to each load with various methods. Fairness index gives idea of fair loss allocation by various methods like PA, QA, GA, PwA and G-RA. Fairness index for each allocation method is given in last row of table no-I. We can observe from table no-I that fairness index is high i.e. 0.9773 for PwA method. G-RA method has fairness index of 0.957. QA has worst fairness index i. e 0.54. QA method was applied with consideration of quadratic expression of loss. Out of PA, QA and GA, GA has fairness index of 0.861 which shows that GA has better feature of transmission loss allocation among existing methods.

Table no-II and figure-II are test cases same various methods at bus no-14 with load combination of 1.5, 2, 3, 4MW with total load of 10.5MW. Results reveal same pattern as of table no-I and figure-I. As we can observe that PwA has fairness index of 0.994 highest among all methods. In existing methods, GA has fairness index of 0.926.

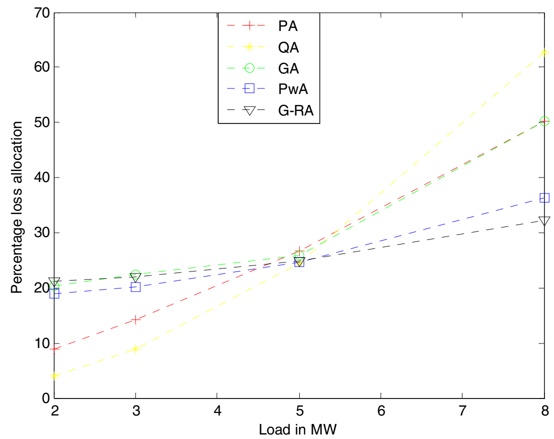

Figure-I: Plot of percentage loss allocation at bus no 13

Table No II: Loss allocation at bus 14

P(MW)	PA	QA	GA	PwA	G-RA
1.5	12.28	7.20	21.85	24.54	22.81
2	17.28	12.80	23.36	23.11	21.65
3	28.64	28.80	26.05	24.07	24.11
4	41.81	51.20	41.81	28.29	31.42
$f(x)$	0.828	0.682	0.926	0.994	0.977

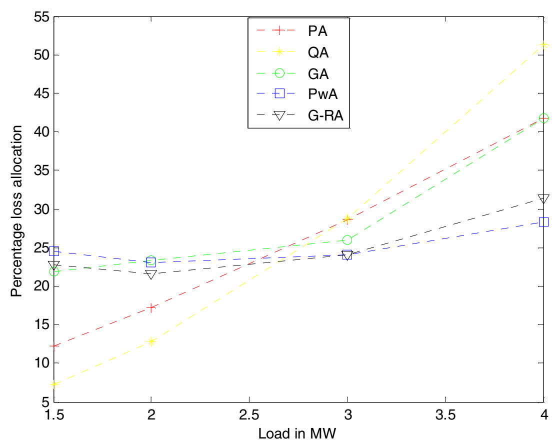

Figure-II: Plot of percentage loss allocation at bus no 14

Table No III: Loss allocation at bus 16

P(MW)	PA	QA	GA	PwA	G-RA
5	9.48	4.49	19.82	24.94	23.79
12	27.29	25.85	26.15	23.01	23.58
18	46.77	58.17	31.24	29.95	30.31
8	16.46	11.49	16.46	22.10	22.31
$f(x)$	0.7591	0.5947	0.9441	0.9855	0.9847

These results verifies the cross- term transmission loss allocation among various load or consumers. Table no-III, IV and V are the results of transmission loss allocation at bus no 16 and 17. On bus no 16 two probable combination of loads has been tested. Figure-III and Figure-IV show the graphical transmission loss allocation at bus no-16 only. Case-I (5, 12, 18 and 8MW) and case-II (6, 14, 20 and 3MW) has been tested at bus no-16.

Table No IV: Loss allocation at bus 16

P(MW)	PA	QA	GA	PwA	G-RA
6	11.06	5.62	21.23	20.01	20.47
14	31.87	30.58	27.99	22.43	23.25
20	52.02	62.40	33.25	31.53	32.18
3	5.045	1.41	5.04	26.03	24.10
$f(x)$	0.646	0.5141	0.8095	0.9708	0.9705

Similar patterns has been observed as in table no-I and II. Among these methods, PwA has maximum fairness index.

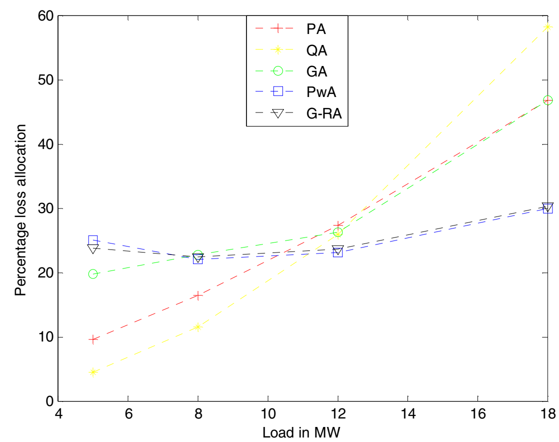

Figure-III: Plot of percentage loss allocation at bus no 16

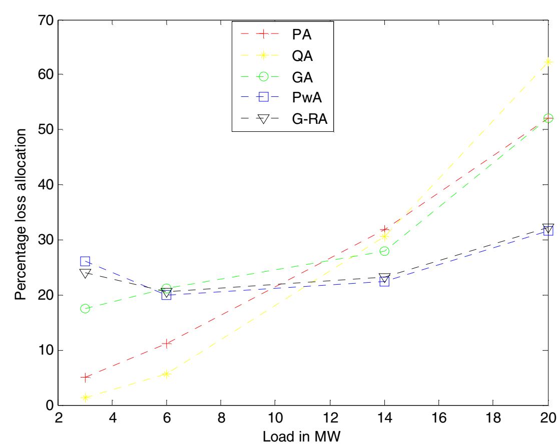

Figure-IV: Plot of percentage loss allocation at bus no 16

Table No V: Loss allocation at bus 17

P(MW)	PA	QA	GA	PwA	G-RA
3	5.10	1.43	17.72	25.01	23.37
5	9.071	3.97	20.36	20.39	20.54
14	32.54	31.11	28.249	22.45	23.21
20	53.29	63.49	53.29	32.16	32.87
$f(x)$	0.6239	0.4983	0.8193	0.9693	0.9661

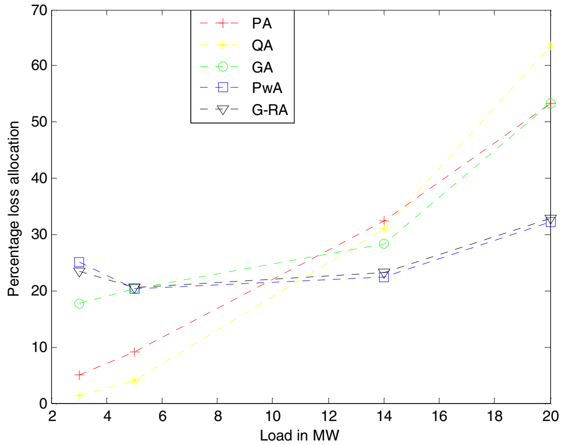

Figure-V: Plot of percentage loss allocation at bus no 17

VI. Conclusion

Proposed methods have tested on IEEE-57 bus system. In IEEE-57 bus network, percentage loss has been allocated to various loads and fairness index evaluated. This loss allocation is done in same network at same bus with different possible load combinations. For different possible loads, proposed algorithm (PwA) shows higher fairness index and aggregate invariance in loss allocation. This was motive behind work done. This paper considers both active and reactive powers for loss calculation. Reactive power was considered with view of accurate transmission loss calculation. Proposed methods gives best cross-term loss allocation among various loads connected to single bus. Fairness allocation has been endorsed by fairness index which is already used by many models of fairness allocation like bandwidth allocation. Proposed method has simple calculation and inherits fair allocation.

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