Compensated Average Modeling for a Buck Converter Control

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***Abstract*—Modeling and control of switched-mode dc-dc converters has occupied a center stage in the field of modern power electronics due to their applications in various industry. Average modeling is an effective tool to analyze dynamic behavior of a converter and are widely accepted in practice because of their simplicity, generality and demonstrated practical utility. However some fundamental questions regarding averaging methodologies still lack satisfactory answers. These unresolved modeling issues are related to their practical validation, inclusion of circuit parasitic. In this paper compensation factors have been proposed for making the model suitable for automatic control.**

***Index Terms*—Buck converter, Duty cycle, Correction factor.**

I. INTRODUCTION

The switched mode dc-dc converters are some of the simplest power electronic circuits which convert one level of electrical voltage into another level by switching action. These converters have received an increasing deal of interest in many areas. This is due to their wide applications like power supplies for personal computers, office equipment, appliance control, telecommunication equipment, DC motor drives, automotive, aircraft, etc. the analysis, control and stabilization of switching converters are the main factors that need to be considered.

Many efforts have been made in the past few decades to model dc-dc converters and several new models have been proposed. These model plays a vital role study the static and dynamic characteristics of the converters as well as to design their regulation control system [9,10]. The so-called averaged models, wherein the effects of fast switching are "averaged" over a switching interval, are most frequently used. Continuous large-signal models are typically non-linear and can be linearized around a desired operating point. Averaged models of dc-dc converters offer several advantages over the switching models. These advantages are determination of transfer-functions, faster simulation of transient response to large-signal perturbations and to allow general-purpose simulators to linearize converters for designing the feedback controller.

Typically a dc-dc converter can operate in two modes. One is the Continuous Conduction Mode (CCM) in which inductor current never falls to zero and the second mode is Discontinuous Conduction Mode (DCM) allowing inductor current to become zero for a portion of switching period. Models for PWM converters, operating in CCM (based on famous state-space averaging technique), were first introduced in 1970's [9,10].

Average modeling is an effective tool to analyze dynamic behavior of a converter and are widely accepted in practice because of their simplicity, generality and demonstrated practical utility. However some fundamental questions regarding averaging methodologies still lack satisfactory answers. These unresolved modeling issues are related to their practical validation, inclusion of circuit parasitic. During analysis it is observed that the transfer function obtained with the help of average modeling in the absence of circuit parasitic some correction factor must be included in order to validate the average model with the actual model.[4]

The dc-dc switching converters are the widely used circuits in electronics systems. They are usually used to obtain a stabilized output voltage from a given input DC voltage which is lower (buck) from that input voltage, or higher (boost) or generic (buck–boost). Each of these circuits is basically composed of transistor and diode making up the switching circuit and inductor and capacitor building the filter circuit. In addition to these, the circuit parameters and various compensation techniques plays a vital role for the purpose of controlling the output parameters [1-3].

Voltage-mode control and Current-mode control are two commonly used control schemes to regulate the output voltage of dc-dc converters. Both control schemes have been widely used in low-voltage low-power switch-mode dc-dc converters integrated circuit design in industry. Feedback loop method automatically maintains a precise output voltage regardless of variation in input voltage and load conditions.

Currently, there exist many different approaches that have been proposed for the PWM switching control design, e.g., state space averaging methods PID control, optimal control, sliding mode control and fuzzy control etc[4,5].

A computationally efficient method for obtaining the small-signal model of a switch mode power converter operating in the continuous conduction mode is presented. To apply this proposed technique, the only analytical work involved is the derivation of the two switched circuits of a given switch mode power converter in state-space representation form, and the developed computer simulation program provides a good approximated small-signal model in the form of open-loop Bode plots. The proposed modeling technique is applied to the buck converter and the derived model is compared with the small-signal model obtained using the state space averaging technique. To further verify the validity of the proposed modeling technique, a 24W buck converter was built to demonstrate the effectiveness of the proposed modeling technique[10].

II. DC-DC BUCK CONVERTER

The buck converter circuit converts a higher dc input voltage to lower dc output voltage. The basic buck dc-dc converter topology is shown

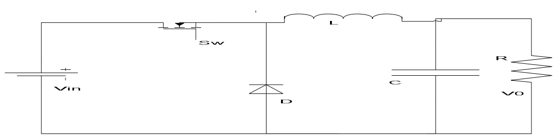
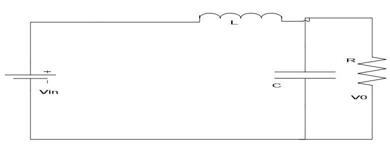
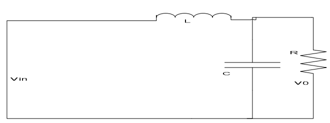


Fig.1. DC-DC Buck Converter



ON State



OFF State

Fig. 2. Operating Modes of Buck Converter

It consists of a controlled switch (S_W), an uncontrolled switch (D), an inductor (L), a Capacitor (C), and a load resistance(R).

The first sub-circuit state is when the switch is turned on, diode is reverse biased and inductor current flows through the switch. When the switch (S_W) is on and D is reverse biased, the dynamics of inductor current (I_L) and the capacitor voltage (V_C) are

$$\frac{dI_L(t)}{dt} = -\frac{1}{L}\times\left(V_0 - V_{in}\right) \tag{1}$$

$$\frac{dV_C}{dt} = \frac{1}{C} i_C(t) \tag{2}$$

The second sub-circuit state is when the switch is turned off and current freewheels through the diode. When the switch Sw is off and D is forward biased, the dynamics of the circuit are

$$\frac{dI_L(t)}{dt} = -\frac{1}{L}\times V_0 \tag{3}$$

$$\frac{dV_C}{dt} = \frac{1}{C} i_C(t) \tag{4}$$

The operation of dc-dc converters can be classified by the continuity of inductor current flow. So dc-dc converter has two different modes of operation that are

(a) Continuous conduction mode (CCM)

(b) Discontinuous conduction mode (DCM)

A converter can be designed in any mode of operation according to the desired value. When the inductor current flow is continuous of charge and discharge during a switching period, it is called Continuous Conduction Mode (CCM).When the inductor current has an interval of time staying at zero with no charge and discharge then it is said to be working in Discontinuous Conduction Mode (DCM) operation and the waveform of inductor current.

III. DETERMINING AVERAGE MODEL OF A BUCK CONVERTER IN OPEN LOOP MODE

To demonstrate the performance of the proposed dc-dc buck converter, in MATLAB/Simulink with the parameters as given in Table 2.1. A constant voltage source of 20 V is input to the converter with R load having the value R =5Ω. The complete model consists of a voltage source, a linear load, a voltage source PWM converter.

TABLE 1. PARAMETERS USED FOR THE BUCK CONVERTER

Switching frequency	f_s=10 kHz
Input voltage	V_g=20 V
Duty Cycle	D=0.5
Inductance	L=83.3 mH
Capacitance	C=25 μF
Load resistance	R_L=5 Ω

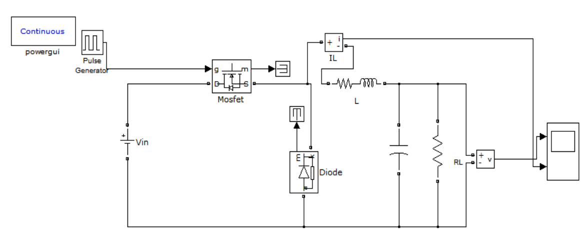


Fig. 3. Simulink/MATLAB model of Buck Converter

When Switch is close for time dT_s, the converter reduces to a linear circuit whose equations can be written in the following state-space form:

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) \tag{5}$$

$$y(t) = C_1 x(t) + E_1 u(t) \tag{6}$$

The matrices A_1, B_1, C_1 and E_1 describes the network connections during the first subinterval.

$$A_1 = \begin{bmatrix} -0.0180 & -12.0048 \\ 4000 & -8000 \end{bmatrix} \quad (7)$$

$$B_1 = \begin{bmatrix} 12.0048 \\ 0 \end{bmatrix} \quad (8)$$

$$C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad (9)$$

$$E_1 = [0] \quad (10)$$

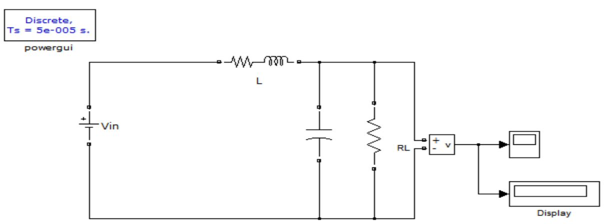


Fig. 4. Buck Converter when Switch is Close & Diode Open

During the second subinterval, the converter reduces to another linear circuit, whose state-space equations are:

$$\dot{x}(t) = A_2 x(t) + B_2 u(t) \quad (11)$$

$$y(t) = C_2 x(t) + E_2 u(t) \quad (12)$$

$$A_2 = \begin{bmatrix} -0.0180 & -12.0048 \\ 4000 & -8000 \end{bmatrix} \quad (13)$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (14)$$

$$C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad (15)$$

$$E_2 = [0] \quad (16)$$

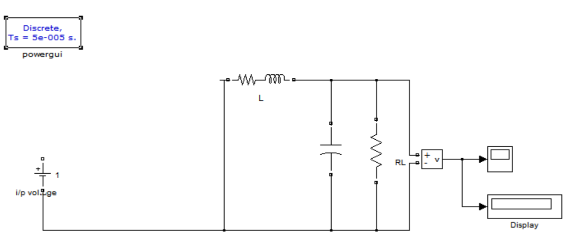


Fig. 5. Buck Converter when Switch is Open & Diode Close

The matrices A_2, B_2, C_2 and E_2 describe the network connections during the second subinterval of length (1-d)Ts. A continuous-time low-frequency nonlinear model is obtained by averaging (5),(6),(11) and (12) over an entire switching cycle Ts and is represented in the following form:

$$\dot{\bar{x}}(t) = A(d)\ \bar{x}(t) + B(d)\ u(t) \quad (17)$$

$$y(t) = C(d)\ \bar{x}(t) + E(d)\ u(t) \quad (18)$$

Where x is the state vector and $\bar{x}(t)$ denotes the average of x over an entire switching period T_s. The corresponding averaged state matrices are given below:

$$\left.\begin{aligned} A(d) &= dA_1 + (1-d)A_2 \\ B(d) &= dB_1 + (1-d)B_2 \\ C(d) &= dC_1 + (1-d)C_2 \\ E(d) &= dE_1 + (1-d)E_2 \end{aligned}\right\} \quad (19)$$

$$A(d) = \begin{bmatrix} -15/833 & -10000/833 \\ 40000 & -8000 \end{bmatrix} \quad (20)$$

$$B(d) = \begin{bmatrix} (10000 * d)/833 \\ 0 \end{bmatrix} \quad (21)$$

$$C(d) = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad (22)$$

$$E(d) = [0] \quad (23)$$

The above representation is nonlinear since system matrices depend on the control signal d(t). It is assumed that the natural frequencies of the converter network are much smaller than the switching frequency. This assumption coincides with the small-ripple approximation, and is usually satisfied in well-designed converters. The converter waveforms are expressed as dc values plus small ac variations, as follows:

$$\left.\begin{aligned} x(t) &= X + \tilde{x}(t) \\ u(t) &= U + \tilde{u}(t) \\ y(t) &= Y + \tilde{y}(t) \\ d(t) &= D + \tilde{d}(t) \end{aligned}\right\} \quad (24)$$

The state-space averaged model that describes the dc converter waveforms is:

$$\left.\begin{aligned} 0 &= AX + BU \\ Y &= CX + EU \end{aligned}\right\} \quad (25)$$

The state equations of the small-signal model can be represented as:

$$\left.\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + B\tilde{u}(t) + G\tilde{d}(t) \\ \tilde{y}(t) &= C\tilde{x}(t) + E\tilde{u}(t) + H\tilde{d}(t) \end{aligned}\right\} \quad (26)$$

Where A,B,C,E,G and H are constant matrices which depend on the converter topology. In general, for continuous conduction mode G and H are given as below:

$$\left.\begin{aligned} G &= (A_1 - A_2)X + (B_1 - B_2)U \\ H &= (C_1 - C_2)X + (E_1 - E_2)U \end{aligned}\right\} \quad (27)$$

$$G = \begin{bmatrix} (10000 * u)/833 \\ 0 \end{bmatrix} \quad (28)$$

$$H = [0] \quad (29)$$

The equation (26) describes how small-signal variations in the input vector and duty cycle excite variations in the state and output vectors. By taking Laplace transform of (26), all small-signal transfer functions can now be derived. These transfer function are given below in their general form:

$$\left.\begin{aligned} \left.\frac{X(s)}{U(s)}\right|_{\tilde{d}=0} &= (sI-A)^{-1}B \\ \left.\frac{X(s)}{D(s)}\right|_{\tilde{u}=0} &= (sI-A)^{-1}G \\ \left.\frac{Y(s)}{U(s)}\right|_{\tilde{d}=0} &= C(sI-A)^{-1}B+E \\ \left.\frac{Y(s)}{D(s)}\right|_{\tilde{u}=0} &= C(sI-A)^{-1}G+H \end{aligned}\right\} \qquad (30)$$

Therefore, control-to-output transfer function from equation (30) can be given as

$$\left.\frac{Y(s)}{D(s)}\right|_{\tilde{u}=0} = \frac{(400000000 * \mathrm{u})}{(833 * \mathrm{s}^\wedge 2 + 6664015 * \mathrm{s} + 400120000)} \qquad (31)$$

In general we can write from equation (31) as

$$\left.\frac{Y(s)}{D(s)}\right|_{\tilde{u}=0} = \frac{(480192.1 * \mathrm{u})}{(\mathrm{s}^\wedge 2 + 8000.02 * \mathrm{s} + 480336.1)} \qquad (32)$$

After finding the transfer function we introduce it in the original circuit as shown in figure and then plot the various wave forms for different duty cycle.

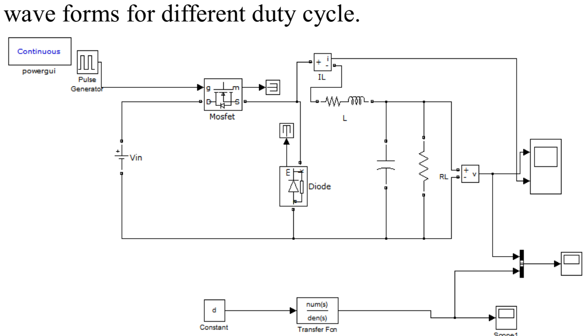


Fig. 6. Buck Converter simulink model with average model transfer function

For d=0.3

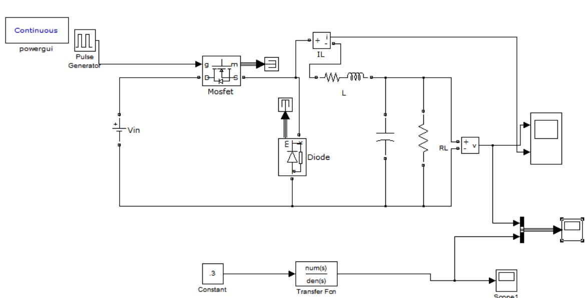


Fig. 7. Buck Converter circuit model with average model transfer function having duty cycle 0.3

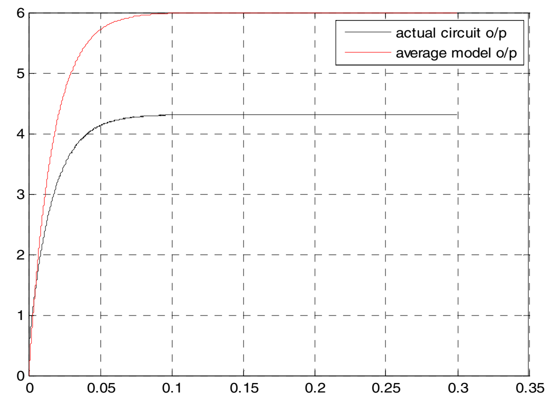


Fig. 8. O/P Waveform of actual circuit and average model over duty cycle of 0.3

After application of various correction factors we obtain the table as

TABLE 2. CORRECTION FACTOR DETERMINED FOR VARIOUS DUTY CYCLE

Circuit Parameters: L=83.3 mH, C=25 µF, f=10 KHz					
Duty Cycle	Actual Circuit (V)	Average Model (V)	C.F.	ΔV_c (V)	ΔI_L (A)
0.1	0.6	2	0.3	0.0013	0.0035
0.2	2.15	4	0.542	0.003	0.005
0.3	4.3	6	0.719	0.003	0.006
0.4	6.8	8	0.851	0.004	0.009
0.5	9.4	10	0.949	0.004	0.005
0.6	12.15	12	1.014	0.004	0.008
0.7	14.6	14	1.044	0.003	0.005
0.8	16.7	16	1.045	0.0024	0.003
0.9	18.4	18	1.021	0.0012	0.002

From the above table of correction factor corresponding to the duty cycle we generate a general equation for correction factor which can be given as:

$$C.F. = -1.046d^4 + 2.8337d^3 - 4.3161d^2 + 3.494d - 0.0079$$

This modifies the state-space average model given by eqn. (32) of the converter as:

$$\left.\frac{Y(s)}{D(s)}\right|_{\tilde{u}=0} = \frac{(480192.1\,\mathrm{u})CF}{(\mathrm{s}^2 + 8000.02\,\mathrm{s} + 480336.1)} \qquad (33)$$

IV. RESULTS & DISCUSSIONS

With the use of correction factor along with the transfer function obtained by the small signal model we had seen that for any duty cycle the output waveform of actual circuit and average model is almost same. It is verified by taking any arbitrary duty cycle and after the comparison of the waveform

obtained from the actual circuit and from the average model.
For d=0.67

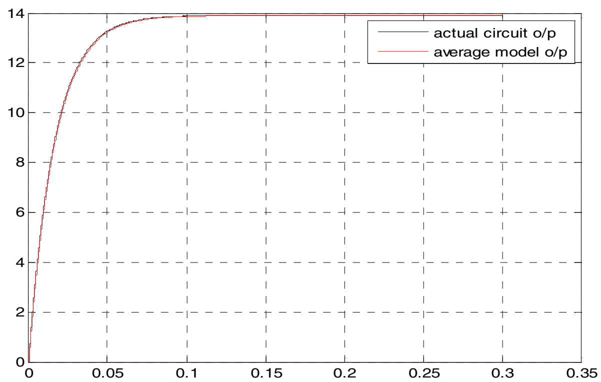


Fig. 9. O/P waveform of actual circuit and average model with duty cycle of 0.67

For d=0.34

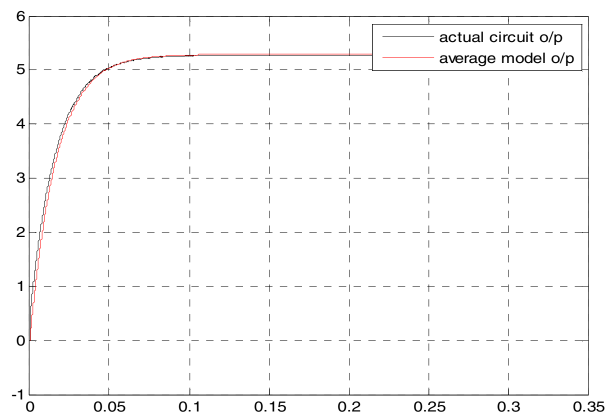


Fig. 10. O/P waveform of actual circuit and average model with duty cycle of 0.34

TABLE 3. ACTUAL CIRCUIT AND AVERAGE MODEL VOLTAGE FOR ANY ARBITRARY DUTY CYCLE

Duty Cycle	V(Actual Circuit)	V(Average Model)
0.31	4.548 V	4.557 V
0.46	8.4 V	8.41 V
0.59	11.9 V	11.88 V
0.67	13.9 V	13.89 V
74	15.5 V	15.51 V

V. Conclusion

It's concluded that with the use of proposed correction factor (which is a 4th order duty cycle polynomial) the output voltage of actual circuit and the average model become same. For any duty cycle it is observed that the average model with the correction factor is correct. The paper has validated the effectiveness of the correction factor by carrying out closed loop control on the developed model.

References

[1] M. Gopal (2002), *Control Systems: Principles and Design*, Second edition, TMH Publication, ISBN 0-07-048289-6.

[2] M.E. Van Valkenburg (2001), *Network Analysis*, Third edition, PHI Publication, ISBN 81-203-0156-0.

[3] Erickson R.W., Maksimovi D. (2000), *Fundamentals of power electronics*, Second edition, Kluwer Academic Publishers, ISBN 0-7923-7270-0

[4] Muhammad Usman Iftikhar, "Investigation of DC-DC Converter Modeling from the Perspective of Control and Input-Filter Influence," A *Doctoral Thesis*, 2008, Department of Energy and Power Systems of Ecole Supérieure d'Electricité (Supélec), France.

[5] Shen-Yaur chen and Jin-Jia Chen, "Study of the effect and design criteria of the input filter for buck converters with peak current-mode control using a novel system block diagram," *IEEE transactions on industrial electronics*, vol.55, No.8, august 2008.

[6] R. D. Middlebrook, "Modeling a current-programmed buck regulator," in Proc. *IEEE Application Power Electronics,* Conf., Rec., 1987, pp. 3–13.

[7] Rim, C.T.; Joung, G.B.; ChoG.-H," A state-space modeling of nonideal DC-DC converters," *IEEE conference on power electronics,* vol.2, 11-14 April 1988.

[8] R. D. Middlebrook and S. Cuk, "A general unified approach to modeling switching-converter power stages," International Journal of Electronics, 42(6): pp. 521–550, Jun. 1977.

[9] R. D. Middlebrook and S. Cuk, "Modeling and analysis methods for dc-to-dc switching converters," in *IEEE Int. Semiconductor Power Converter Conf.*, pp. 90–111, Mar. 1977 Record.

[10] Lee, J.W. ; Nowicki, E. ; Park, G.C., "A computational small-signal modeling technique for switch mode power converter,"Telecommunications EnergyConference (INTELEC), 32nd International, pp. 1 - 7, 6-10 June 2010.