Fractional Order System Identification and Controller Design Using PSO

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***Abstract*— This paper presents a Particle Swarm Optimization (PSO) based method to identify the fractional order transfer function of an industrial heating furnace system using its experimental data. Also the PSO technique for design of PID controller is proposed for the same system with its frequency and time domain analysis. For tuning of the PID controller in this system, PSO technique has been investigated and the performance is analyzed using MATLAB. A fitness function is proposed which provides the most suitable controller design.**

Keywords*— *Fractional order plant, Fitness Function, Heating Furnace, Particle Swarm Optimization.

I. Introduction

There is an increasing interest in dynamic systems of non-integer orders which are employed in extending the order of derivatives and integrals from integer to non-integer order due to its advantages of robustness and improved controllability [1]. In the literature, people often use the term “fractional order calculus” or “fractional order dynamic system” where “fractional” actually means “non-integer”. The advantages of fractional order control in modeling and control design have motivated renewed interest in various applications of fractional order control [2-4]. In industries, the process control techniques have made great advancements and strategies like adaptive control, fuzzy control, neural control and neuro-fuzzy control have also gained popularity [5-9]. However, the most common and widely used method is feedback strategy with Proportional-Integral-Derivative (PID) controller.

PID controller is widely used in the industries because of its simple structure, robust performance and wide range of operating applications, but it is difficult to tune the gains of PID controllers properly because many industrial process are often of high order, contains time delays, and nonlinearities [5-10]. Over the years, many heuristic methods have been proposed for the tuning of PID controllers. The classical method that employs the tuning rules was proposed by Ziegler and Nichols. In general, it is quite hard to determine optimal PID parameters with the Ziegler-Nichols formula in many industrial plants [5-7], so it is highly desired to enhance the capabilities of PID controller. The neural network based method is also available to tune the controller but have disadvantages like, training data availability [9-11]. To overcome these problems nature inspired optimization techniques can be used and Particle Swarm Optimization (PSO) is one of them. PSO was first introduced by Kennedy and Eberhart. It is one of the modern heuristic algorithms that as developed through simulation of a simplified social system, and has been found to be robust in finding solution of continuous nonlinear optimization problems [12-16]. The PSO technique can generate a high-quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods [15-18], by selecting proper fitness function, Naka et al. [19] have presented the application of hybrid PSO method for solving the practical distribution state estimation problem. This study develops the PSO-PID controller to search optimal PID parameters. The PSO method is an excellent optimization methodology and a promising approach for solving the optimal PID controller parameters problem.

A fitness function in time domain is proposed in this paper for evaluating the performance of an integer/fractional order system that employs PID controller. Firstly, PSO is employed for determining integer/fractional system and secondly, PSO is again employing for obtaining gains of PID controller. The integer/fractional order dynamic system modeling, control and performance analysis is carried out on MATLAB platform in this paper to establish the effectiveness of proposed function.

II. Design of integer/fractional order system

The fractional order control lead to more adequate modeling and more robust control performance which is a generalization of classical integral order control theory. Ho. et al. [20] have presented a simple tuning formula for the design of PID controllers. Some MATLAB tools for simulating the fractional order dynamic system modeling, control and filtering can be found in [21].

Fractional order systems could model various real systems more adequately than integer order ones and thus provide an excellent modeling tool in describing many dynamical processes. It is discussed in [22], that these fractional order models are used to achieve excellent performance.

An example of a heating furnace is described by Podlubny et al. [23] and the same has been used in this paper. In the following text the system determination from measured value of step-response of a similar heating furnace system is carried out (shown in Fig. 1).

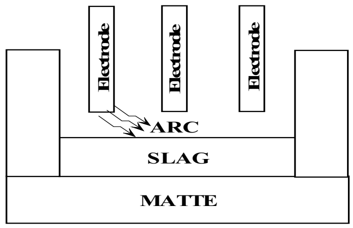

Fig. 1. Physical Model of Electric Arc Furnace

The set of measured values of step response yi* (i=0, 1, …M) can be used to obtain the transfer function of this real experimental heating furnace. Two models have been developed for this system from the step-response: integer order model and fractional order model.

To obtained integer order model it is assumed that this heating furnace system can be described by a second order differential equation, given as:

$$a_2 y_I''(t) + a_1 y_I'(t) + a_0 y_I(t) = u(t) \tag{1}$$

To have minimum difference between the measured step response value of yi* and step response of this modeled system y_{Ii} from eq. (1), the criterion function is described as:

$$Q = \frac{1}{M+1}\sum_{i=0}^{M} |(y_i^* - y_{Ii})| \tag{2}$$

where, y_{Ii} is the output of the model at the point of ith measurement. M is the number of data points in step response. Firstly, the objective is to obtain value of a_2, a_1, a_0 such that Q is minimum. In this case the obtained minimal value of Q employing PSO is 0.0012 and a_2, a_1 and a_0 are:

$$a_2 = 10000, a_1 = 3164, a_0 = 0.9956$$

This gives the integer order transfer function

$$G_I = \frac{1}{10000s^2 + 3164s + 0.9956} \tag{3}$$

Now, the second model can be obtained assuming that the same system can be described by the three-term fractional differential written as:

$$b_2 y_F^{\alpha}(t) + b_1 y_F^{\beta}(t) + b_0 y_F(t) = u(t) \tag{4}$$

PSO technique is employed to determine b_2, b_1, b_0 and α, β so that the step response is as close as possible to measured value of step response. In this case, the values found using eq. (2) criteria are α = 1.0888 and β=0.001, $b_2 = 6484.1, b_1 = 0.001, b_0 = 1.017$ giving the value Q=0.0006984 for the criterion and transfer function is:

$$G_F = \frac{1}{6484s^{1.0888} + 0.001s^{0.001} + 1.017} \tag{5}$$

Fig. 2(a) shows the experimentally measured output data (yi*) of the system along with integer order(y_I) and fractional order identified system output(y_F). The expanded view is shown in Fig. 2(b). Fig. 3 shows the block diagram of process which is heating furnace with controller.

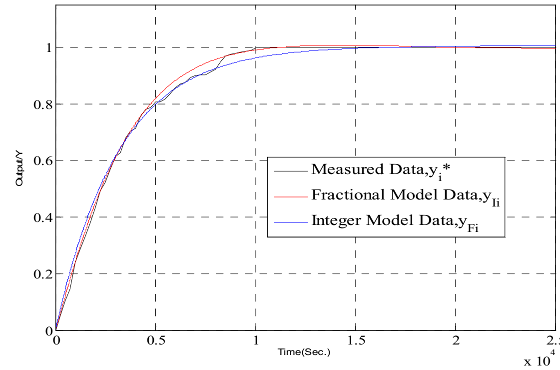

Fig. 2(a). Plot showing measured output of an industrial system along with step response obtained treating the system as integer order and fractional order.

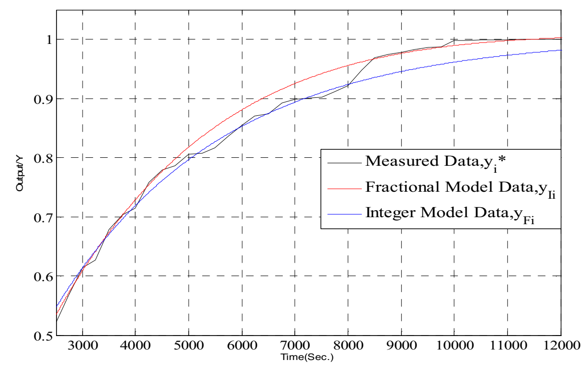

Fig. 2(b). Expanded view of Fig. 1 showing step response in range 2500 to 12000 Sec. for clearity

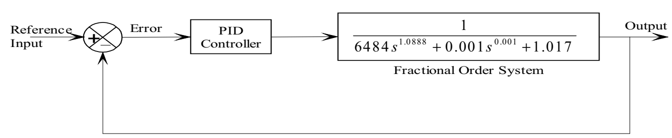

Fig. 3. Block diagram of fractional order model of heating furnace with PID controller

III. PSO APPORACH FOR SYSTEM IDENTIFICATION AND CONTROLLER TUNING

PSO is a nature inspired technique based around the study of collective behavior in decentralized, self-organized systems called Swarm intelligence (SI). These systems are typically made up of a population of simple agents interacting locally with one another and also with their environment, local interactions between of agents results to the emergence of a desired global behavior [24-26].

The individual in a swarm is treated as a volume-less particle in multidimensional continuous search space. The position in search space represents a potential solution of optimization problem, and the velocity of an individual particle determines the direction and step of search. The particle moves in search space at definite velocity between the steps which is dynamically adjusted according to its own moving experience and that of the companions. It constantly adjust its approach direction and velocity by tracing the best position found so far by particles themselves and that of the whole swarm, which forms positive feedback of swarm optimization. Every particle swarm tracks the two best current positions, moves to better region gradually, and finally arrives to the best position in the continuous search space.

In PSO, Let us consider that a swarm consists of N number of particles moving around in a d dimensional search space represented as:

$$X_i = (x_{i1}, x_{i2}, \dots \dots \dots, x_{id}) \quad (6)$$

Also assume that the previous best solution (p_{best}) for each individual is represented as:

$$P_i = (p_{i1}, p_{i2}, \dots \dots \dots, p_{id}) \quad (7)$$

Also the current velocity for each individual determined using x_{id} and p_{id} be:

$$V_i = (v_{i1}, v_{i2}, \dots \dots \dots, v_{id}) \quad (8)$$

Finally, the best solution of whole swarm (g_{best}) also called global best shall be found from the personal best of individuals represented as:

$$P_g = (p_{g1}, p_{g2}, \dots \dots \dots, p_{gd}) \quad (9)$$

At each time step, each particle is guided by(p_{best}) and (g_{best}) locations. A suitable fitness function value depends on dimensions of search space evaluates the performance of particles to decide if the best fitting solution is achieved [27]. The algorithm of particle swarm optimization and the flow chart (Fig. 4) is employed in as follows.

A. Algorithm of PSO:

Step 1: Initialization: The velocity and position of all particles are randomly set within pre-defined ranges in first step

Step 2: Fitness Function: The fitness is evaluated for each particle of the swarm.

Step 3: Velocity Updation: At each iteration, the velocity of particles is updated as

$$v_{id} = v_{id} + C_1 * rand1 * (p_{gd} - x_{id}) + C_2 * rand2 * (p_{gd} - x_{id}) \quad (10)$$

Where C_1 and C_2 are two positive constants, called cognitive and social learning rate respectively. The constant rand1 is a random /function in the range [0, 1]. Since the formula (eqn. 10) of PSO lacks velocity control mechanism, it has a poor ability to search a narrow solution, a parameter called inertia weight (w) is required in the original equation for balancing the global and local search (eqn. 11).

$$w = (w_{max} - w_{min}) * \frac{(Iter_{max} - Iter_{now})}{Iter_{max}} + w_{min} \quad (11)$$

Therefore, the velocity equation will shall be:

$$v_{id} = w * v_{id} + C_1 * rand1 * (p_{gd} - x_{id}) + C_2 * rand2 * (p_{gd} - x_{id}) \quad (12)$$

Step 4: Position Updating: The positions of all particles are updated between successive iterations of unit time interval as:

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (13)$$

After updating, these values should be checked and limited in the allowed range.

Step 5: Memory Updating: Updated p_{best} and g_{best} are given as:

$$p_{best} = p_i \quad \text{if } F(p_i) < F(p_{best}) \quad (14)$$

$$g_{best} = g_i \quad \text{if } F(g_i) < F(g_{best}) \quad (15)$$

where F (x) is the objective function subject to minimize.

Step 6: Stopping Criteria: The algorithm repeats Steps 2 to 5 till a certain stopping condition is met. It shall stop after a pre-defined number of iterations or a failure to make progress for a certain number of iterations. Once terminated, the algorithm shall report the value of p_{best} , g_{best} as its solution [28].

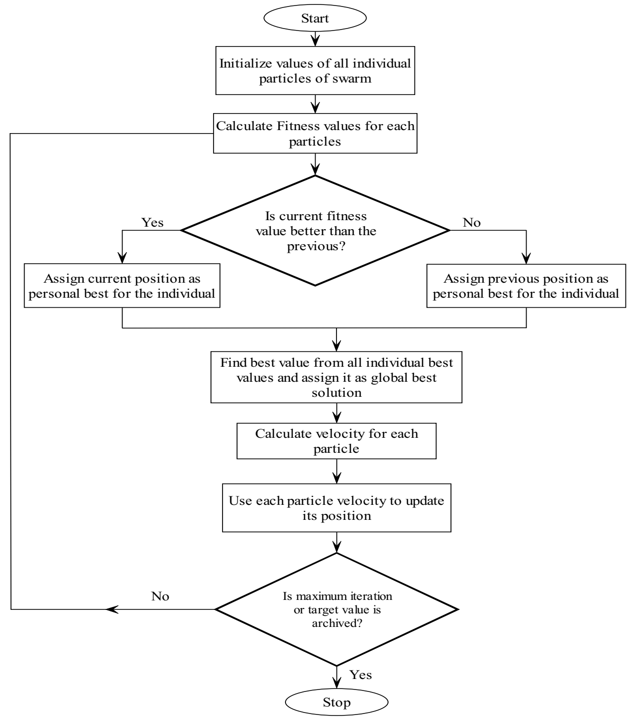

Fig. 4. Flow chart of PSO Algorithm

In this paper a PSO algorithm has been developed and the various constants employed can be seen in Table 1. The PSO algorithm shall be employed for following purpose (i) To obtain the values of constant $a_2, a_1$and a_0 of eqn.(1). (ii) To obtain the values of b_2, b_1, b_0, α and β of eqn. (4). (iii) Three parameters of PID controller [$K_p \quad K_d \;\; K_i$] have to be tuned such that it gives the best output results. In other words we have to optimize all the parameters of the PID for best results.

IV. Fitness Function

The performance of the designed system with designed PID controller can be determined by a fitness function. The PSO algorithm require a suitable fitness function for its working and therefore for the designed integer/fractional order plant four performance indices have been considered to define its fitness function which are peak overshoot, steady state error, settling time and rise time. However the contribution of these indices on fitness function is depended on a scaling factor that depends upon the system and experience of the designer. For this type of system the best performance is the point where the performance indices have the minimum value. To get best step response from the designed system the performance indices dependent fitness function is proposed as:

$$F = 1 - (e^{-\gamma_1 M_p} * e^{-\gamma_2 e_{ss}} * e^{-\gamma_3 T_s} * e^{-\gamma_4 T_r}) \quad (16)$$

Where F: Fitness function
M_p: Peak Overshoot
e_{ss}: Steady State error
T_s: Settling Time
T_r: Rise Time
γ_1,γ_2,γ_3 and γ_4: Scaling Factor

V. VALIDATION OF DESIGNED SYSTEM

Fig. 5-7 shows the process and simulation model developed for system identification and tuning of PID controller. The validation is carried out on MATLAB platform using three steps- STEP I, STEP II and STEP III.

STEP I: In this step the values of b_2, b_1, b_0, α and β of eqn. (4) are found using PSO algorithm (Fig. 5)

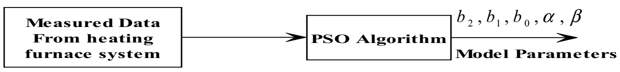

Fig. 5. Calculation of Heating Furnace Model parameters using PSO

STEP II: In this step, the closed loop controlled system is developed with a PID controller but K_p, K_d and K_i are not known. The PSO has been employed to generate these values and step response is obtained to obtain performance indices $\%Mp$, t_r, e_{ss}, t_s. Accordingly the value of fitness function is obtained using eqn. (16) and optimum value of K_p, K_d and K_i are determined by PSO technique for the system. These values now employed for experimental system.

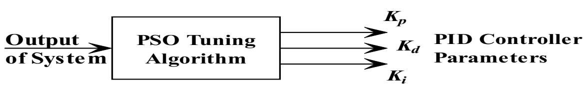

Fig. 6. Calculation of PID controller parameters using PSO

STEP III: The values obtained in STEP 1 and STEP 2 are fed to the system and its output is determined as shown in Fig 7. The saturation block have been used to limit the i/p to 1.5, which is considered the overload capacity of the input.

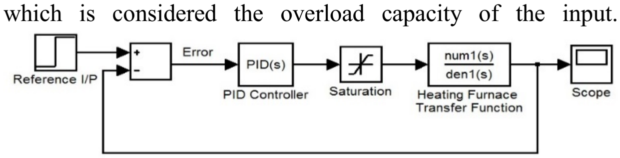

Fig. 7. Simulink model with PID controller

PSO has been used to optimize the values of K_p, K_d and K_i and the values of $[K_p, K_d, K_i]$ obtained from it for integer order system and fractional order system are [10000, 8532.4286, 0.971] and [10000, 210.88, 0.10] respectively. Fig. 8(a) and Fig. 8(b) shows the step response of integer order system and fractional order system with the designed PSO-PID controller with reference value one and 0.6 respectively.

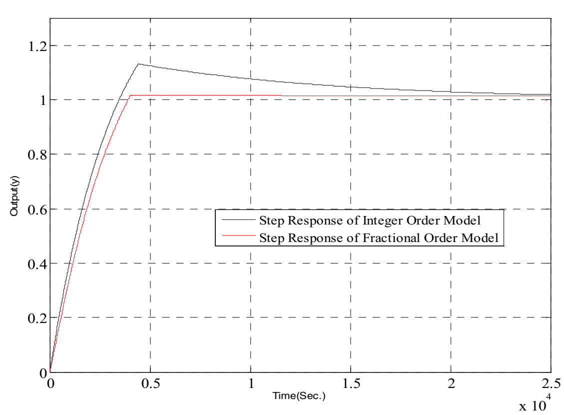

Fig. 8(a). Step Response of with PSO tuned PID controller for integer and fractional order system

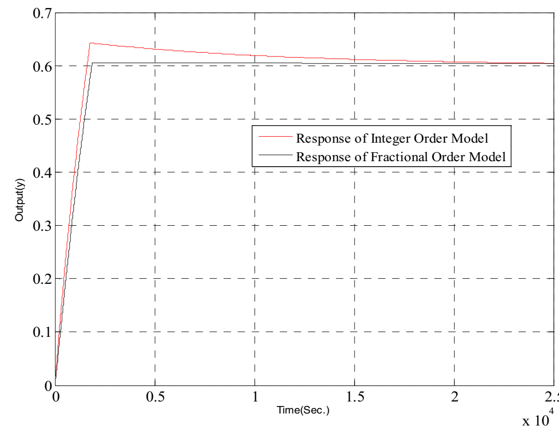

Fig. 8(b). Response of with PSO tuned PID controller for integer and fractional order system with reference value 0.6

The value of various time domain performance parameters for PSO-PID controller like rise time, %peak overshoot, and settling time for integer order model with PSO-PID controller are 3472Sec., 13.2% and 23660Sec, respectively, the same parameters for fractional order model with PSO-PID controller are 3923Sec., 1.6% and 3923Sec.

For frequency domain analysis the Bode plot of the open loop transfer function of the closed loop system is drawn Fig. 9(a) and 9(b) shows the Bode plot of the integer order system having PSO-PID controller and fractional order system having PSO-PID controller respectively. It is observed from Bode plots that both the systems are stable with phase margin 60.0° and 83.8° respectively, and also the bandwidth of fractional order system is better as compared to integer order system as fractional order system has higher gain crossover frequency which is 1.49rad/sec. as compared to integer order system for which it is 1.17rad/sec.

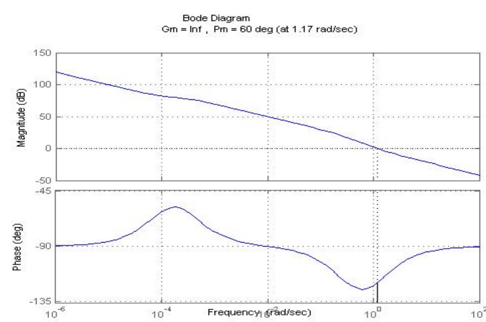

Fig. 9(a). BODE plot of integer order system with PSO-PID controller

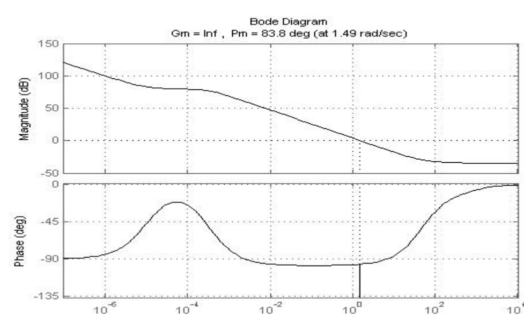

Fig. 9(b). BODE plot of fractional order system with PSO-PID controller

All the parameters and other details of these developed models are given in Table 1.

TABLE 1. DESCRIPTION OF MODEL PARAMETERS

1	Integer order system parameters obtained by PSO:	a_2= 10000, a_1= 3164, $a_0 = 0.9956$, $Q = 0.0012$
2	Fractional order system parameters obtained by PSO:	b_2= 6484.1, b_1= 0.001,$b_0 = 1.017$, α = 1.0888, β =0.001, $Q = 0.0006984$
3	Parameters for criterion function in eq. (2):	M = 101
4	Parameters employed in PSO algorithm:	N = 50, C_1 =1.5, C_2 = 2.5, w_{max} = 0.9, w_{min} = 0.1, $Iter_{max}$ =20
5	Constants employed in fitness function:	γ_1= 0.0005, γ_2=0.0005, γ_3 =0.05 and γ_4=0.05
6	PID controller parameters and performance indices obtained by PSO for Integer order model	K_p=10000, K_d = 8532.4286, Ki = 0.971, $\%M_p$=13.2%, t_r=3472sec. e_{ss}=0 and t_s= 23660sec., Phase Margin= 60.0° at (1.17rad/sec.)
7	PID controller parameters and performance indices obtained by PSO for Fractional order model	K_p=10000, K_d = 210.88, Ki = 0.1 $\%M_p$=1.6%, t_r=3923sec. e_{ss}=0 and t_s= 3923sec., Phase Margin= 83.8° at (1.49rad/sec.)

It can be noticed that fitness of fractional order system is better and its response fits more closely to experimental results, as seen in Fig. 2. The closed loop response (Fig. 8(a)) and stability (Fig. 9 and 10) are also superior in case of fractional order model as compared to integer order model.

VI. CONCLUSION

This paper dealt with the analysis of a heat furnace which have been modeled as integer as well as fractional order industrial system. The transfer function is evaluated from the measured experimental data using the PSO technique. PID controller parameters are also determined by PSO to get best response. The simulated results shows the effectiveness of PSO technique and proposed fitness function in industrial process control. Also, the fractional order model gives a more appropriate representation of system compared to integer order model.

It is concluded that the PSO technique can be used for determining the transfer function of any system from its open loop response. Moreover, the PSO can be employed for determining controller parameters to get best performance indices. The fitness function based on performance indices (eqn. 16) for PID tuning is applicable for any mathematical model of a control system.

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