Fuzzy Warehouse Inventory Model for Items with Imperfect Quality under Trade Credit Policy and Inflationary Conditions

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***Abstract*— The objective of this study is to derive the optimal retailer's replenishment policy for imperfect quality items in fuzzy environment. The effect of permissible delay in payments is also considered. It is assumed that production rate depends on the demand rate. Due to different preservation facilities, we consider that the deterioration rate is time dependent in own warehouse (O.W) and Wei-bull distribution in rented warehouse (R.W). The ordering cost, purchasing cost, holding cost and selling price are taken as fuzzy numbers.**

Keywords--warehouse, Deterioration, Permissible delay in payment, Inflation, supply chain, Fuzzy.

I. Introduction

Computational Intelligence is a new modern way to deal with the complex problems of the real world applications to which the traditional methodologies and approaches are ineffectual. It plays a crucial role in inventory modeling with supply chain system. In inventory modeling the rate of deterioration of goods is viewed as an exogenous variable, which is not subjected to control

Generally in all inventory models, it is assumed that all the items produced are of perfect quality but it is not true. These models provide some general understanding of the behavior of inventory system under different assumptions, but are not capable of representing real life situations. Using Fuzzy set theory, instead of traditional probability theory produces more accurate results. Here is a brief review of the literature. Rosenblatt and Lee [1] assumed that the defective items could be reworked instantaneously at a cost and presence of defective products motivates smaller lot-sizes, Chung and Hou [2] extended this model in which the shortages are permitted. Salameh and Jaber [3] assumed the defective percentage as the random variable and defective items could be sold as a single batch at a discounted price prior to receiving the next shipment, Hayek and Salameh [4] assumed that the defective percentage is distributed uniformly and the shortages are permitted. Chiu [5] assumed that not all products can be reprocessed into the good-quality ones.

Fuzzy set theory was introduced by Zadeh. It received significant consideration among researchers. To estimate different inventory parameters as crisp or stochastic is a difficult task for the retailers. Nowadays, it is not easy to decide the exact cost parameters such as holding cost, ordering cost, purchase cost and selling price etc. These parameters may contain some uncertain values. In these circumstances, it is better to model these parameters as fuzzy. Kacprzyk and staniewski [9] investigated long-term inventory policy-making using fuzzy decision models for a multi-stage inventory planning problem. Park [10] developed an EOQ model with the ordering and inventory holding costs being trapezoidal fuzzy numbers. Roy and Maiti [11] transformed an EOQ model into non-linear programming problem after fuzzifying the objective function and the available storage space. Mandal and Maiti [12] developed a non-linear fuzzy modeling for a multi-item EOQ model with imprecise storage space and number of production run constraints where few input parameters were fuzzified. Yao and Chang [13] developed an EOQ model with the total demand and the unit holding cost being triangular fuzzy numbers. They used the signed distance and the centroid as de fuzzification methods. Vijayan and Kumaran [14,15] proposed inventory models with partial backorders and lost sales and fuzzy stock-out periods, Mahata and Mahata [16] developed a fuzzy EOQ model for deteriorating items under retailer partial trade credit financing in a supply chain. We have developed an inventory model incorporating the real life situations that will help the retailers to survive in the market. We assume production rate as being dependent on the demand rate and the two warehouses have different deterioration rates. The ordering cost, holding cost, purchase cost, deterioration cost and selling price are taken as triangular fuzzy numbers. The annual total cost for the retailer is de-fuzzified using Graded Mean Integration Representation Method. The objective of this study is to derive the retailer's optimal replenishment policy that minimizes the total relevant inventory costs in fuzzy environment. Finally, some numerical examples for illustration are provided and sensitivity analysis on parameters is made.

II. Modelling of EPQ-based Inventory Problemwith Fuzzy Variables

A. Assumptions and notations

The mathematical models of the two-warehouse inventory problems are based on the following assumptions:

1. Production rate is greater than demand rate. Also, it is linear combination of on-hand inventory and demand rate

i.e . $P(t)=[I(t)+bD(t)](1-e^{-dt})$

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1. Demand rate is exponentially an increasing function of time .i.e. $D(t)=\mu e^{\lambda t}, 0 \leqslant \lambda \leqslant 1$.
2. Deterioration is taken as time dependent for O.W, while, Wei-bull distribution for R.W.
3. Planning Horizon is finite.
4. Model is considered for imperfect items and inflation is also taken in this model.
5. Shortages are not permitted.
6. Lead time is zero, and no replenishment or repair of deteriorated items is made during
a given cycle.
7. A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.
8. The owned warehouse (O.W) has a fixed capacity of W units, and the rented warehouse (R.W) has unlimited capacity.
9. The goods of the O.W are consumed only after consuming the goods kept in R.W.
10. The unit inventory costs (including holding cost) per unit time in R.W are higher than those in O.W.
11. The supplier provides the retailer a permissible delay of payments. During the trade credit period the account is not settled, the revenue is deposited in an interest bearing account. At the end of the permissible delay, the retailer pays off the items ordered, and starts to pay the interest charged on the items in stock.

In addition, the following notations are used throughout this paper.

$D(t)=\mu e^{\lambda t}, 0 \leqslant \lambda \leqslant 1$: Demand rate increases with time, where μ is the initial demand rate.

$P(t)=[I(t)+bD(t)](1-e^{-dt}),\ 0 \leqslant d \leqslant 1, b > 1$: Production rate

r : Inflation rate.

d : Rate of imperfect production.

W : Fixed capacity of O.W.

C_S : Set up cost per production run.

C_{RW} : Holding cost per unit inventory held in R.W. per unit time.

C_{OW} : Holding cost per unit inventory held in O.W. per unit time.

C_D : Deterioration cost per unit time.

$\widetilde{c}_S$: Fuzzy set up cost per unit time.

$\widetilde{c}_{RW}$: Fuzzy holding cost per unit inventory held in R.W. per unit time.

$\widetilde{c}_{OW}$: Fuzzy holding cost per unit inventory held in O.W. per unit time.

$\widetilde{c}_D$: Fuzzy deterioration cost per unit time.

$\widetilde{s}$: Fuzzy unit selling price.

$\widetilde{c}$: Fuzzy unit purchase cost.

$I_{O_1}(t)$: Inventory level in O.W. at time t with $t \in [0, t_1]$.

$I_{R_2}(t)$: Inventory level in R.W. at time t with $t \in [t_1, t_2]$.

$I_{R_3}(t)$: Inventory level in R.W. at time t with $t \in [t_2, t_3]$.

$I_{O_4}(t)$: Inventory level in O.W. at time t with $t \in [t_3, T]$.

$I_{O_5}(t)$: Inventory level in O.W. at time t with $t \in [t_1, t_3]$.

t_1, t_2 : Production period for O.W and R.W.

t_3, T : Non-Production period.

T : Total cycle time.

M : Retailer's trade credit period offered by supplier in years.

s : Unit selling price.

c : Unit purchase cost.

I_e Interest which can be earned per \$ per year.

I_C : Interest charges per \$ in stocks per year by the supplier.

$TC_{i,}, i=1,2,3$:

Total costs, includes
(a) Ordering cost (b) Stock holding cost
(c) Deteriorating cost (d) Interest payable cost (e) Interest earned.where T^*, t_3^* are the optimal solutions and TC^* is the minimum total cost.

B. *The mathematical model formulation and its analysis*

The model begins as follows: Initially, the inventory level is zero. The production starts at time t = 0, and items accumulate from 0 up to W units in O.W .and in t_1 units of time. After time t_1 any production quantity exceeding W will be stored in R.W. After this production stopped and the inventory level in R.W. begins to decrease at t_2 and will reach 0 units at t_3 because of demand and deterioration. The inventory level in O.W. comes to decrease at t_1 and then falls below W at t_2+t_3 due to deterioration. But, during $[t_3, T]$, the inventory is depleted due to both demand and deterioration.

The differential equations stating the inventory levels within the cycle are given as follows:

$$\frac{dI_{O_1}(t)}{dt}+\theta I_{O_1}(t)=P(t)-D(t),\ 0 \leqslant t \leqslant t_1 \qquad (1)$$

$$\frac{dI_{R_2}(t)}{dt}+\theta I_{R_2}(t)=P(t)-D(t),\ t_1 \leqslant t \leqslant t_2 \qquad (2)$$

$$\frac{dI_{R_3}(t)}{dt}+\theta I_{R_3}(t)=-D(t),\ t_2 \leqslant t \leqslant t_3 \qquad (3)$$

$$\frac{dI_{O_4}(t)}{dt}+\theta I_{O_4}(t)=-D(t),\ t_3 \leqslant t \leqslant T \qquad (4)$$

$$\frac{dI_{O_5}(t)}{dt}+\theta I_{O_5}(t)=0,\ t_1 \leqslant t \leqslant t_3 \qquad (5)$$

With the boundary conditions

$$I_{O_1}(0)=0, I_{R_2}(t_1)=0, I_{R_3}(t_3)=0, I_{O_4}(T)=0, I_{O_5}(t_1)=W$$

respectively, the above equation can be solved successively as follows:

The solutions to (1) - (5) are

$$I_{O_1}(t)=(b\quad 1)\mu\; \frac{dt^2}{2}+\frac{\lambda dt^3}{3}\quad \frac{(\theta\quad d)dt^4}{4}\quad \mu\; t+\frac{\lambda t^2}{2}\quad (\theta\quad d)\frac{t^3}{2} \tag{6}$$

$$I_{R_2}(t)=(b\quad 1)\mu\; \frac{d\left(t^2\quad t_1^2\right)}{2}\quad \frac{\alpha d t^{\beta}\left(t^2\quad t_1^2\right)}{2}+\frac{\left(t^3\quad t_1^3\right)\lambda d}{3}\quad \mu\; \left(t\quad t_1\right)\quad \alpha t^{\beta}\left(t\quad t_1\right)+\frac{\lambda\left(t^2\quad t_1^2\right)}{2}\quad \left(t^2\quad t_1^2\right)\frac{\lambda\alpha t^{\beta}}{2}$$

(7)

$$I_{R_3}(t)=\mu\; \left(t_3\quad t\right)\quad \alpha t^{\beta}\left(t_3\quad t\right)+\frac{\lambda}{2}\left(t_3^{\,2}\quad t^2\right)\quad \frac{\lambda\alpha t^{\beta}}{2}\left(t_3^{\,2}\quad t^2\right) \tag{8}$$

$$I_{O_4}(t)=\mu\; (T\quad t)\quad \frac{\theta t^2}{2}(T\quad t)+\frac{\lambda}{2}\left(T^2\quad t^2\right)\quad \frac{\lambda\theta t^2}{4}\left(T^2\quad t^2\right) \tag{9}$$

$$I_{O_5}(t)=W\; e^{\frac{\theta}{2}\left(t_1^{\,2}\quad t^2\right)} \tag{10}$$

Based on the assumptions and description of the model, the total annual relevent costs, TC, include the following elements:

(1)The present value of ordering cost = $C_S\, e^{-rt}$.

(2) The present values of the inventory holding costs in R.W. and O.W. are

$$I_{RW}=c_{RW}\left(\int_{t_1}^{t_2} I_{R_2}(t)e^{-rt}dt+\int_{t_2}^{t_3} I_{R_3}(t)e^{-rt}dt\right)=$$

$$c_{RW}\left(\mu\left((b-1)\left(\frac{d}{2}\left(\frac{t_2^{\,3}}{3}+\frac{2t_1^{\,3}}{3}\right)-\frac{dr}{3}\left(\frac{t_2^{\,4}}{4}+\frac{t_1^{\,4}}{2}\right)+\frac{d\lambda}{3}\left(\frac{t_2^{\,4}}{4}+\frac{3t_1^{\,4}}{4}\right)-\frac{rd\lambda}{3}\left(\frac{t_2^{\,5}}{5}+\frac{3t_1^{\,5}}{10}\right)\right)\right)\right.$$

$$\left.-\left(\left(\frac{t_1^{\,2}}{2}+\frac{t_3^{\,2}}{2}\right)-r\left(\frac{t_1^{\,3}}{6}+\frac{t_3^{\,3}}{6}\right)+\frac{\lambda}{2}\left(\frac{2t_1^{\,3}}{3}+\frac{2t_3^{\,3}}{3}\right)-\frac{\lambda r}{2}\left(\frac{t_1^{\,4}}{4}+\frac{t_3^{\,4}}{2}\right)\right)\right)$$

$$I_{OW}=c_{OW}\left(\int_0^{t_1} I_{O_1}(t)e^{-rt}dt+\int_{t_1}^{t_3} I_{O_5}(t)e^{-rt}dt+\int_{t_3}^{T} I_{O_4}(t)e^{-rt}dt\right)$$

$$=c_{OW}\left((b-1)\mu\left(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\right)-\mu\left(\frac{t_1^{\,2}}{2}-\frac{T^2}{2}-\frac{t_3^{\,2}}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\right)\right.$$

$$\left.+W\left(\left(t_3-t_1\right)-\frac{r}{2}\left(t_3^{\,2}-t_1^{\,2}\right)+\frac{\theta t_1^{\,3}}{2}\right)\right)$$

(3)The present value of the inventory deterioration cost is

$$I_D=c_D\left(\int_0^{t_1}\theta I_{O_1}(t)e^{-rt}dt+\int_{t_1}^{t_3}\theta I_{O_5}(t)e^{-rt}dt+\int_{t_3}^{T}\theta I_{O_4}(t)e^{-rt}dt+\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}I_{R_2}(t)e^{-rt}dt\right.$$

$$\left.+\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_{R_3}(t)e^{-rt}dt\right)$$

$$=c_D\left(\theta\mu\left((b-1)\left(\frac{dt_1^4}{8}+\frac{d\lambda t_1^{\,5}}{15}+\frac{drt_1^5}{10}\right)-\left(\frac{t_1^{\,3}}{3}+\frac{\lambda t_1^{\,4}}{8}+\frac{rt_1^{\,4}}{4}\right)\right)\right.$$

$$+W\theta\left(\frac{t_3^{\,2}}{2}-\frac{t_1^{\,2}}{2}+\frac{\theta t_3^{\,4}}{8}-\frac{\theta t_1^{\,4}}{8}-\frac{rt_3^{\,3}}{3}+\frac{rt_1^{\,3}}{3}\right)+\theta\mu\left(\left(\frac{-t_3^{\,3}}{6}-\frac{T^3}{3}\right)-r\left(\frac{-t_3^{\,4}}{12}-\frac{T^4}{4}\right)\right.$$

$$\left.+\frac{\lambda r}{2}\left(\frac{-2t_3^{\,5}}{15}-\frac{T^5}{5}\right)\right)+\mu^2\alpha^2\beta^2\left((b-1)d\left(\frac{t_2^{\,\beta+2}}{2(\beta+2)}+\frac{t_1^{\,\beta+2}}{2\beta(\beta+2)}\right)\right)-\left(\frac{t_2^{\,\beta+1}}{\beta+1}+\frac{t_1^{\,\beta+1}}{\beta(\beta+1)}\right)\Bigg)$$

$$\left(\left(\left(\frac{t_2^{\,\beta+1}}{(\beta+1)}+\frac{t_3^{\,\beta+1}}{\beta(\beta+1)}\right)-r\left(\frac{t_2^{\,\beta+1}}{(\beta+2)}+\frac{t_3^{\,\beta+2}}{(\beta+2)(\beta+1)}\right)\right)\right)\Bigg)$$

Next, Based on the parameter values t_3, M, T, there are three cases to be explored.

Case 1: $M\le t_3\le T$, In this case, interest payable is

$$IC_1=cI_C\left(\int_M^{t_3} I_{R_3}(t)dt+\int_M^{t_3} I_{O_5}(t)dt+\int_{t_3}^{T} I_{O_4}(t)dt\right)$$

$$=cI_C\left(\left(\frac{t_3^{\,2}}{2}-\frac{\alpha t_3^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{\lambda t_3^{\,3}}{3}-\frac{\lambda\alpha t_3^{\,\beta+3}}{(\beta+1)(\beta+3)}-\left(t_3M-\frac{M^2}{2}\right)-\alpha\left(\frac{t_3M^{\beta+1}}{(\beta+1)}\right.\right.\right.$$

$$\frac{\lambda}{2}\quad t_3^{\,2}M\quad \frac{M^3}{3}\quad +\frac{\lambda\alpha}{2}\quad t_3^{\,2}\frac{M^{\beta+1}}{\beta+1}\quad \frac{M^{\beta+3}}{\beta+3}\quad +W\;\left(t_3\quad M\right)+\frac{\theta t_1^{\,2}}{2}\left(M\quad t_3\right)+\frac{\theta}{6}\left(t_3^{\,3}\quad M^3\right)$$

$$+\mu\frac{T^2}{2}\quad\frac{\theta T^4}{24}+\frac{\lambda T^3}{3}\quad\frac{\lambda\theta T^5}{30}\quad Tt_3+\frac{t_3^{\,2}}{2}+\frac{\theta T t_3^{\,3}}{6}\quad\frac{\theta t_3^{\,4}}{8}\quad\frac{\lambda}{2}\;T^2t_3\quad\frac{t_3^{\,3}}{3}\quad+\frac{\lambda\theta}{4}\frac{T^2t_3^{\,3}}{3}\quad\frac{t_3^{\,5}}{5}$$

Case 2: $t_3\le M\le T$,In this case, interest payable is

$$IC_2=cI_C\int_M^T I_{O_4}(t)dt$$

$$=cI_C\left(\frac{T^2}{2}-\frac{\theta T^4}{24}+\frac{\lambda T^3}{3}-\frac{\lambda\theta T^5}{30}-TM+\frac{M^2}{2}+\frac{\theta TM^3}{6}-\frac{\theta M^4}{8}\right.$$

$$\left.-\frac{\lambda}{2}\left(T^2M-\frac{M^3}{3}\right)+\frac{\lambda\theta}{4}\left(\frac{T^2M^3}{3}-\frac{M^5}{5}\right)\right)$$

Case 3: $M\le T$, In this case, no interest charges are paid for the items

$$IC_3=0$$

On the other hand, the retailer accumulates revenue in an account that earns I_e per dollar per year staring from t_3 to T. As a result, the interest earned is given as follows:

There are two cases as follows:

Case1: $M\le T$, In this case, the interest earned is

$$IE_1 = sI_e\int_0^M Dtdt = \frac{sI_e\mu}{\lambda^2 T}\left(M\lambda e^{\lambda M} - e^{\lambda M} + 1\right)$$

Case2: $M \le T$, In this case, the interest earned is

$$IE_2 = sI_e\left(\int_0^T Dtdt + DT(M-T)\right)$$

$$= sI_e\mu\left(\left(\frac{\lambda T e^{\lambda T} - e^{\lambda T} + 1}{\lambda^2}\right) + \mu e^{\lambda t} T(M-T)\right)$$

Therefore, the annual total relevant costs for the retailer can be expressed as:

TC(t_3,T) = ordering cost + stock holding cost in RW+ stock holding cost in OW + deteriorating cost +interest payable cost – interest earned.

That is,

$$TC(t_3,T) = \begin{cases} TC_1, \text{ if } M \le t_3 \le T \\ TC_2, \text{ if } t_3 \le M \le T \\ TC_3, \text{ if } M \le T \end{cases} \quad (11)$$

III. FUZZY MODEL

In order to develop the model in a fuzzy environment, we consider the costs as the triangular fuzzy numbers where $C_S^{\sim}$ = (C_S-δ_{S1}, C_S , C_S+δ_{S2}), $C_{RW}^{\sim}$ = (C_{RW}-δ_{RW1}, C_{RW} , C_{RW}+δ_{RW2}), $C_{OW}^{\sim}$ = (C_{OW} - δ_{OW1}, C_{OW} , C_{OW} + δ_{OW2}), $C_D^{\sim}$ = (C_D - δ_{SD1}, C_D , C_D + δ_{D2}),

$C^{\sim}$ = (C- δ_{C1}, C, C+δ_{C2}), $S^{\sim}$ = (S - δ_{S1}, S , S + δ_{S2}),where δ_{S1}, δ_{S2}, δ_{RW1}, δ_{RW2}, δ_{OW1}, δ_{OW2},

δ_{SD1}, δ_{D2}, δ_{C1}, δ_{C2} ,δ_{S1}, δ_{S2} are determined by the decision maker based on the uncertainty of the problem .Thus , the costs are considered as the fuzzy numbers with membership function. For any T > 0, we get fuzzy annual total variable costs

$TC_1(t_3,T)^{\sim}=X_{11}\otimes C_S^{\sim}\oplus X_{12}\otimes C_{RW}^{\sim}\oplus X_{13}\otimes C_{OW}^{\sim}\oplus X_{14}\otimes C_D^{\sim}\oplus C^{\sim}\otimes X_{15}$

$TC_2(t_3,T)^{\sim}=X_{21}\otimes C_S^{\sim}\oplus X_{22}\otimes C_{RW}^{\sim}\oplus X_{23}\otimes C_{OW}^{\sim}\oplus X_{24}\otimes C_D^{\sim}\oplus C^{\sim}\otimes X_{25}\otimes C^{\sim}\Theta S^{\sim}\otimes X_{26}$

$TC_3(t_3,T)^{\sim}=X_{31}\otimes C_S^{\sim}\oplus X_{32}\otimes C_{RW}^{\sim}\oplus X_{33}\otimes C_{OW}^{\sim}\oplus X_{34}\otimes C_D^{\sim}\Theta S^{\sim}\otimes X_{36}$

Where ⊗,⊕ and Θ are the fuzzy arithmetical operations under the functional principle.

We defuzzify the fuzzy annual total variable costs and obtain the graded mean integration representation as follows:

$P(TC_1(t_3,T)^{\sim}) = X_{11}F_1 + X_{12}F_2 + X_{13}F_3 + X_{14}F_4 + X_{15}F_5$

$P(TC_1(t_3,T)^{\sim}) = X_{21}F_1 + X_{22}F_2 + X_{23}F_3 + X_{24}F_4 + X_{25}F_5$

$P(TC_1(t_3,T)^{\sim}) = X_{31}F_1 + X_{32}F_2 + X_{33}F_3 + X_{34}F_4 + X_{35}F_5$

Where

$F_1 = (C_{S1}+4\,C_S+C_{S2})/6$; $F_2 = (C_{RW1}+4\,C_{RW}+C_{RW2})/6$;

$F_3 = (C_{OW1}+4\,C_{OW}+C_{OW2})/6$;

$F_4 = (C_{D1}+4\,C_D+C_{D2})/6$; $F_5 = (S_1+4\,S+S_2)/6$

$$-\left(\left(\frac{t_1^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_1^3}{6}+\frac{t_3^3}{6}\right)+\frac{\lambda}{2}\left(\frac{2t_1^3}{3}+\frac{2t_3^3}{3}\right)-\frac{\lambda r}{2}\left(\frac{t_1^4}{4}+\frac{t_3^4}{2}\right)\right)\right)$$

$$X_{13}=X_{23}=X_{33}=\frac{1}{T}\left((b-1)\mu\left(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\right)-\mu\left(\frac{t_1^2}{2}-\frac{T^2}{2}-\frac{t_3^2}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\right)\right.$$

$$\left.+W\left((t_3-t_1)-\frac{r}{2}(t_3^2-t_1^2)+\frac{\theta t_1^3}{2}\right)\right)$$

$$X_{14}=X_{24}=X_{34}=\frac{1}{T}\left(\left(\theta\mu\left((b-1)\left(\frac{dt_1^4}{8}+\frac{d\lambda t_1^5}{15}+\frac{drt_1^5}{10}\right)\right.\right.\right.$$

$$\left.-\left(\frac{t_1^3}{3}+\frac{\lambda t_1^4}{8}+\frac{rt_1^4}{4}\right)\right)+W\theta\left(\frac{t_3^2}{2}-\frac{t_1^2}{2}+\frac{\theta t_3^4}{8}-\frac{\theta t_1^4}{8}-\frac{rt_3^3}{3}+\frac{rt_1^3}{3}\right)+\theta\mu\left(\left(\frac{-t_3^3}{6}-\frac{T^3}{3}\right)\right.$$

$$-r\left(\frac{-t_3^4}{12}-\frac{T^4}{4}\right)+\frac{\lambda r}{2}\left(\frac{-2t_3^5}{15}-\frac{T^5}{5}\right)\right)+\mu^2\alpha^2\beta^2\left(\left((b-1)d\left(\frac{t_2^{\beta+2}}{2(\beta+2)}+\frac{t_1^{\beta+2}}{2\beta(\beta+2)}\right)\right)\right.$$

$$\left.\left.\left.-\left(\frac{t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta(\beta+1)}\right)\right)-r\left(\frac{t_2^{\beta+1}}{(\beta+2)}+\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}\right)\right)\right)\right)$$

$$X_{15}=\frac{I_c}{T}\left(\left(\frac{t_3^2}{2}-\frac{\alpha t_3^{\beta+2}}{(\beta+1)(\beta+2)}\right.\right.$$

$$+\frac{\lambda t_3^3}{3}-\frac{\lambda\alpha t_3^{\beta+3}}{(\beta+1)(\beta+3)}-\left(t_3M-\frac{M^2}{2}\right)+\alpha\left(\frac{t_3M^{\beta+1}}{\beta+1}-\frac{M^{\beta+2}}{\beta+2}\right)-\frac{\lambda}{2}\left(t_3^2M-\frac{M^3}{3}\right)$$

$$+\frac{\lambda\alpha}{2}\left(\frac{t_3^2M^{\beta+1}}{\beta+1}-\frac{M^{\beta+3}}{\beta+3}\right)\right)+W\left((t_3-M)+\frac{\theta t_1^2}{2}(M-t_3)+\frac{\theta}{6}\left(t_3^3-M^3\right)\right)$$

$$+\mu\left(\frac{T^2}{2}-\frac{\theta T^4}{24}+\frac{\lambda T^3}{3}-\frac{\lambda\theta T^5}{30}-Tt_3+\frac{t_3^2}{2}+\frac{\theta Tt_3^3}{6}-\frac{\theta t_3^4}{8}-\frac{\lambda}{2}\left(T^2t_3-\frac{t_3^3}{3}\right)\right.$$

$$\left.\left.+\frac{\lambda\theta}{4}\left(\frac{T^2t_3^3}{3}-\frac{t_3^5}{5}\right)\right)\right)\right)$$

$$X_{25}=\frac{I_c\mu}{T}\left(\frac{T^2}{2}-\frac{\theta T^4}{24}+\frac{\lambda T^3}{3}-\frac{\lambda\theta T^5}{30}-TM+\frac{M^2}{2}\right.$$

$$\left.+\frac{\theta T M^3}{6}-\frac{\theta M^4}{8}-\frac{\lambda}{2}\left(T^2M-\frac{M^3}{3}\right)+\frac{\lambda\theta}{4}\left(T^2\frac{M^3}{3}-\frac{M^5}{5}\right)\right)$$

$$X_{26}=\frac{sI_e\mu}{\lambda^2T}\left(M\lambda e^{\lambda M}-e^{\lambda M}+1\right))$$

$$X_{36}=\frac{I_e\mu}{T}\left(\left(\frac{Te^{\lambda T}\lambda-e^{\lambda T}+1}{\lambda^2}\right)+\mu e^{\lambda T}T(M-T)\right)$$

$$X_{11}=X_{21}=X_{31}=\frac{1}{T}\left(e^{-rt}\right)$$

IV. NUMERICAL EXAMPLE

To illustrate the above model described, we considered the following data in fuzzy sense on the basis of the previous study.

Let $C_S^{\sim}$ = (9.9, 10, 10.1), $\theta=0.01$, r =1, $\lambda=2$, μ=20,
$C_D^{\sim}$ = (1.9, 2, 2.1), I_C = 0.15, M = 0.25, α =0.06, β=0.01, d =0.1, t_1=0.5, t_2 =0.6, b = 5,
$C_{RW}^{\sim}$ = (0.5, 0.6, 0.7), W =500, $C_{OW}^{\sim}$ = (1.1, 1.2, 1.3),
$C^{\sim}$ = (0.1, 0.2, 0.3),
s ~ = (0.0,0.1,0.2) in appropriate unit.
It can be found that the optimal solutions is t_3^*=0.8, T^*=264992 and TC^*= 51.1379.

A. Sensitivity analysis

To study the effect of change of the parameter, sensitivity analysis is performed considering the numerical example given above .Sensitivity analysis is performed by changing the parameters and taking one parameter at a time, taking the remaining parameter at original value on the basis of data given in example. The results are shown in Table 1 for permissible delay in payment (Trade credit) by using software Mathematica 5.2.

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B. Result difference:

Z = Total Cost in Crisp Sense, $\tilde{Z}$ = Total Cost in Fuzzy Sense. Then

$$\frac{|Z-\tilde{Z}|}{Z}*100;\ \frac{|50.6496-51.1379|}{50.6496}*100=0.9\%$$

TABLE I. SENSITIVITY ANALYSIS TABLE

Parameter	% change	T^*	TC^*
M	0.26	2.64224	51.5654
	0.27	2.6346	51.9901
	0.28	2.62698	52.4118
	0.29	2.61939	52.8308
α	0.07	2.64958	50.9971
	0.08	2.64947	50.8291
	0.09	2.64975	50.6305
	0.1	2.6507	50.3961
β	0.02	2.64993	51.1428
	0.03	2.64994	51.1477
	0.04	2.64996	51.1524
	0.05	2.64997	51.1571
μ	22	1.68713	70.9504
	24	1.73694	69.1958
	26	1.80075	66.0144
	28	1.34198	85.1272
W	550	2.67866	52.9379
	600	2.75795	53.6072
	700	2.65586	60.6593
	750	2.6148	64.6759
θ	0.02	2.80684	44.45
	0.03	1.69699	70.4
	0.04	1.93343	63.8411
	0.05	2.06369	59.6782

From the above table, following inferences can be observed :

(1) When retailer's warehouse capacity W is increasing, the optimal replenishment cycle time T^* will decrease but the relevant total costs TC^* will increase. This implies that the retailer can order quantity less frequent to reduce costs when the retailer owns larger storage space.

(2) It can be found that the optimal cycle time will decrease when the initial demand rate μ is increasing. It means that the retailer will order less quantity to take the benefits of the trade credit more frequently.

(3) On increasing the deterioration parameter for Owned Warehouse, then T^* decreases, TC^* decreases. On increasing the deterioration parameter for Rented Warehouse, then T^* increases and total cost slightly increases decreases

(4) When retailer's warehouse capacity W is increasing, the optimal replenishment cycle time T^* will increases but the relevant total costs TC^* will decreases. This implies that the retailer can order quantity less frequent to reduce costs when the retailer owns larger storage space.

(5) It can be found that the optimal cycle time will decrease when the initial demand rate μ is increasing. It means that the retailer will order less quantity to take the benefits of the trade credit more frequently.

(6) On increasing the deterioration parameter for Owned Warehouse, then T^*decreases, TC^* decreases.

(7) On increasing the deterioration parameter for R.W, then T^* increases and total cost slightly increases decreases..

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In this paper, a two-warehouse imperfect production inventory model is developed for a manufacturing system. Our aim is to find the optimal replenishment policies for minimizing the total relevant costs. We have built a mathematical model with fuzzy parameters. Triangular membership function are used for all the fuzzy numbers. Numerical examples are also provided to illustrate the proposed model. Sensitivity analysis of the optimal solutions with respect to major parameters is carried out. The problem has been solved by considering fuzzy data as well as crisp data. Both the results have been compared and it is observed that the variation in the results is within the tolerance margin and parameters is in more elastic form then the traditional solution..

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