A Reduced Order Model for AGC System using Routh Approximation Technique

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Abstract- The fast development and usage of small digital computers and processors in the design, analysis and implementation of suitable control strategies has led towards proposing the reduced order modeling of large order physical systems. As most of the models are too complex to be analyzed in its original form, various techniques have been developed in the past to reduce the order of a complex system. The structure of existing power systems is very complex in nature and therefore their resulting mathematical models are of large orders. The analysis of such systems becomes a tedious and difficult job. In this article, a model order reduction technique based on Routh approximation criteria is used to model AGC model of a single area. An AGC model of single area is reduced to third order. The system dynamic response plots for both original and reduced order models are obtained for step input and they are compared. The investigations of these results carried out in the study demonstrate that the complexity of the system is reduced to a great extent. However, the dynamic behavior of all the system states for same step disturbance is within acceptable limit.

Keywords— **Model order reduction, SISO system, Reciprocal transformation, Routh approximation**.

I. INTRODUCTION

Researchers and engineers are often confronted with the analysis, design and synthesis of real life problems. The first step in such studies is the development of a mathematical model which can be a substitute for the real problem.In any modeling task, two often conflicting factors prevail- simplicity and accuracy. On one hand, if a system model is over simplified, presumably for computational effectiveness, inadequate conclusions may be drawn from it. On the other hand, a highly detailed model would lend to a great deal of unnecessary complications. Clearly a mechanism by which a compromise can be made between a complex, more accurate and a simple model is needed.

It is usually possible to describe the dynamics of physical systems by a number of simultaneous differential equations with constant coefficients as:

$$dx/dt = Ax + Bu$$

But for many processes the order of the matrix A is quite large. Various methods have been developed to reduce the order of the matrix so that the computational efforts required in analyzing these systems can be minimized.

There are many advantages of using reduced order model for analyzing the complex and large physical systems. However few of them are as follows:

- To achieve a simpler simulation of the process for better and amenable understanding of its dynamics.
- To reduce the computational effort of obtaining controller structures by deriving control strategies based on reduced order models.
- To obtain a lower dimensional control law for simplifying the structure of the feedback controller.

In power system also it is quite common to study complex processes by simple transfer function, and in most of the systems the order of a system found may be quite large. It would be difficult to work with these complex systems in their original form. In such cases, it is common to study the process of approximating it to a simpler model.

A variety of research articles and books [1] have been published relating the area of model order reduction techniques over the last more than forty years. The techniques proposed in [2-7] are based on the retention of the dominant poles of the system in the reduced model while other states corresponding to non-dominant poles are neglected. The proposed method is very useful from the point of view that the reduced system was stable for stable original system and unstable for the unstable original system. The most of the above approaches involve the computation of the eigenvalues and eigenvectors of the higher order system state matrix. This is computationally very cumbersome, and is known to fail when the eigenvalues of the system are widely separated.

Another approach was appeared in the literature which is based on deriving the reduced model such that the initial time moments of both the system and model are equivalent [8-9]. Further, the method of continued fraction synthesis of reduced models has been found to be a special case of Pade approximation, which for asymptotically stable systems, is equivalent to the time moments methods [10],[11]. These methods are computationally simple but have a very serious disadvantage that the reduced model may be unstable (stable) even though the system is stable (unstable) [12]. The method proposed by Y.Shamash [13] involves the features of both Pade approximation and the modal methods of reduction.

To reduce single input single output (SISO) system is comparatively easier than multi input multi output (MIMO) system. In the year 1975 the Hutton and B. Friedland [14] has presented a new approach to reduce MIMO system. In this approach, the requirement that an approximate to a stable system must be stable is fulfilled. The basic idea underlying this method is to develop the well-known "Routh table" for the original system, and then to construct the approximate model in such a manner that the coefficient of its Routh table agree, up to a given order, with those of the original. By this construction, it is obvious that any Routh approximation of a stable system is stable A derivation for obtaining the Routh approximation has already been presented by Hutton and Friedland. Later the Routh approximation method was modified by Krishnamoorthy and Seshadri [15] , R.Prasad [16] ,[17],[18] and many others. In this paper an AGC model has been considered and its reduced model is obtained by applying Routh approximation technique. The reduced power system model is investigated in the wake of step load disturbance in the system.

II. COMPUTING ROUTH APPROXIMATION

Let the original asymptotically stable system is given as:

$$G(s) = \frac{n_{11} + n_{12}s + n_{13}s^2 + \cdots \ldots \ldots . + n_{1n}s^{n-1}}{d_{11} + d_{12}s + d_{13}s^2 + \cdots \ldots \ldots \ldots . . + d_{1n}s^n} \qquad (1)$$

Equation (1) can be expanded into α, β parameters as:

$$G(s) =$$

$$\beta_1 f_1(s) + \beta_2 f_1(s) f_2(s) + \cdots \ldots + \beta_n f_1(s) f_2(s) \ldots . f_n(s) \quad (2)$$

Where β_i, $i = 1,2, \ldots \ldots \ldots \ldots , n$ are constants and the functions $f_i(s)$ $i = 2,3, \ldots , n$ are defined as:

$$f_i(s) = \cfrac{1}{\alpha_i s + \cfrac{1}{\alpha_{i+1} + \cfrac{1}{\vdots \atop \alpha_{n-1}s + \cfrac{1}{\alpha_n s}}}}$$

The coefficients α_i and β_i can be computed by using denominator and numerator polynomial of original model given by equation (1).
Using α and β coefficients Routh convergent can be obtained to find the reduced order model.

Let $A_r(s)$ and $B_r(s)$ denote the denominator and numerator, respectively, of the r^{th} Routh convergent, then

$$A_1(s) = \alpha_1(s) + 1$$
$$B_1(s) = \beta_1$$
$$A_2(s) = \alpha_1\alpha_2 s^2 + \alpha_2 s + 1$$
$$B_2(s) = \alpha_2\beta_1 s + \beta_2$$

in general form;

$$A_r(s) = \alpha_r s A_{r-1}(s) + A_{r-2}(s) \ldots \ldots \qquad (3)$$

and

$$B_r(s) = \alpha_r s B_{r-1}(s) + B_{r-2}(s) + \beta_r, \quad \ldots . . \qquad (4)$$

$r = 1,2, \ldots \ldots .$

With $A_{-1}(s) = 0$ $B_{-1}(s) = 0$

$A_0(s) = 1$ $B_0(s) = 0$

Routh convergent is an approximation to original model which tends to preserve high frequency behavior.

III. MODEL UNDER CONSIDERATION

For the application point of view an isolated power system model is considered for the investigation. The structure of present day power system is very large and complex in nature therefore a power system model has been considered in the present study to demonstrate the feasibility of the model order reduction technique. The structure and the detailed dynamic models for the isolated system are presented in the ensuing section.

It is a single area isolated power system consisting of non-reheat thermal turbines. The transfer function model is described in Fig.1. A model considered here can be written as follows [19].

$$\frac{d}{dt}\Delta F(t) = -\frac{1}{T_p}\Delta F(t) + \frac{K_p}{T_p}\Delta P_g(t) - \frac{K_p}{T_p}\Delta P_d(t) \qquad (5)$$

$$\frac{d}{dt}\Delta P_g(t) = -\frac{1}{T_T}\Delta P_g(t) + \frac{1}{T_T}\Delta X_g(t) \qquad (6)$$

$$\frac{d}{dt}\Delta X_g(t) = -\frac{1}{RT_G}\Delta F(t) - \frac{1}{T_G}\Delta X_g(t) - \frac{1}{T_G}\Delta E(t) + \frac{1}{T_G}\Delta u(t) \qquad (7)$$

$$\frac{d}{dt}\Delta E(t) = K_E \Delta F(t) \qquad (8)$$

Where:
$\Delta F(t)$= the incremental frequency deviation (Hz);

$\Delta P_g(t)$=incremental change in generator output (p.u.MW)

$\Delta X_g(t)$= incremental change in governor valve position (p.u.MW);

$\Delta E(t)$=incremental change in integral control;

$\Delta P_d(t)$=load disturbance (p.u.MW);

T_G=governer time constant;

T_T=turbine time constant;

T_P=plant model time constant;

K_p=plant gain;

R=speed regulation (Hz p.u.MW^{-1})

K_E=integral control gain

$$\Delta E(t) = K_E \int_0^t \Delta F(t)\, dt$$

The model defined by equations (5)- (8) can be written as

$$\frac{dx}{dt} = Ax(t) + Bu(t) + F\Delta P_d(t) \tag{9}$$

Where

$$x(t) = \left[\Delta F(t), \Delta P_g(t), \Delta X_g(t), \Delta E(t)\right]'$$

We choose the nominal parameters as follows:

$$A = \begin{bmatrix} -0.0665 & 8 & 0 & 0 \\ 0 & -3.663 & 3.663 & 0 \\ -6.86 & 0 & -13.736 & -13.736 \\ 0.6 & 0 & 0 & 0 \end{bmatrix},$$

$$B = [0 \quad 0 \quad 13.736 \quad 0]', \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The original model in transfer function form can be written as:

$$G(s) = \frac{\begin{bmatrix} 0 \\ 0 \\ 13.74 \\ 0 \end{bmatrix} s^3 + \begin{bmatrix} 0 \\ 50.32 \\ 51.23 \\ 0 \end{bmatrix} s^2 + \begin{bmatrix} 402.52 \\ 3.35 \\ 3.35 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 241.5 \end{bmatrix}}{s^4 + 17.47s^3 + 51.5s^2 + 204.4s + 241.5},$$

$$\hat{G}_3(s) = \frac{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 241.5 \end{bmatrix} s^3 + \begin{bmatrix} 402.52 \\ 3.35 \\ 3.35 \\ 0 \end{bmatrix} s^2 + \begin{bmatrix} 0 \\ 50.32 \\ 51.23 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 0 \\ 13.74 \\ 0 \end{bmatrix}}{241.5s^4 + 204.4s^3 + 51.5s^2 + 17.47s + 1}$$

Where $\hat{G}_3(s)$ is reciprocal transformation of G(s).
We indicate with:

ΔP_d as input, $\Delta F(t)$ as output1, $\Delta P_g(t)$ as output2, $\Delta X_g(t)$ as output 3, $\Delta E(t)$ as output 4

IV. CALCULATION OF ALPHA AND BETA TABLE FOR THE SYSTEM

The resultant model is of the order four. This fourth order model is to be reduced to a third order model.

ALPHA TABLE

Alpha coefficients	Original system coefficients of denominator
	$a_0^0 = 241.5 a_2^0 = 51.5 \quad a_4^0 = 1$ $a_0^1 = 204.4 a_2^1 = 17.47$
α_1=1.18	$a_0^2 = 30.885 \quad a_2^2 = 1$
α_2=6.62	$a_0^3 = 10.846 \quad a_2^3 = 0$
α_3= 2.845	$a_0^4 = 1$

BETA TABLE

Beta Coefficients	Original system coefficients of numerator	
	0	0
	402.52	0
β_1=0	0	
β_2=13	-13	
β_3= 0		

Where :

$$\alpha_i = \frac{a_0^{i-1}}{a_0^i} \;\&\; a_0^{i+1} = a_2^{i-1} - \alpha_i a_2^i,$$

$$a_2^{i+1} = a_4^{i-1} - \alpha_i a_4^i \text{………..for} \quad i = 1,2,\dots,n$$

Similarly beta table can be found from second, third and fourth numerator using following relations

$$\beta_1 = \frac{b_0^1}{a_0^1}, \qquad \beta_2 = \frac{b_0^2}{a_0^2}, \qquad \beta_3 = \frac{b_0^3}{a_0^3}$$

Combining values of all beta tables we get:

$$\beta_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.18 \end{bmatrix}; \beta_2 = \begin{bmatrix} 13 \\ 0.11 \\ 0.11 \\ 0 \end{bmatrix}; \beta_3 = \begin{bmatrix} 0 \\ 4.638 \\ 4.72 \\ -1.9 \end{bmatrix}; \beta_4 = \begin{bmatrix} -13 \\ -0.11 \\ 51.12 \\ 0 \end{bmatrix}$$

To get third order approximant

$$\hat{G}_3(s) = \frac{B_3(s)}{A_3(s)}$$

Where $B_3(s)$ and $A_3(s)$ can be calculated from equations (3) and (4).

$$B_3(s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 22.22 \end{bmatrix} s^2 + \begin{bmatrix} 36.98 \\ 0.313 \\ 0.313 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 4.64 \\ 4.72 \\ -0.72 \end{bmatrix}$$

$$A_3(s) = \alpha_1\alpha_2\alpha_3 s^3 + \alpha_2\alpha_3 s^2 + (\alpha_1 + \alpha_3)s + 1$$

$$A_3(s) = 22.22s^3 + 18.83s^2 + 4.025s + 1$$

After applying reciprocal transformation third order approximant is obtained

$$G_3(s) = \frac{\begin{bmatrix} 0 \\ 4.64 \\ 4.72 \\ -0.72 \end{bmatrix} s^2 + \begin{bmatrix} 36.98 \\ 0.313 \\ 0.313 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 22.22 \end{bmatrix}}{s^3 + 4.025s^2 + 18.83s + 22.22}$$

The eigenvalues are computed for both original and reduced order model. These values are given in Table-1

TABLE -1
EIGENVALUES OF ORIGINAL AND THIRD ORDER APPROXIMANT

Original Model	Reduced Order Model
-14.8559 -0.5672 + 3.2656i -0.5672 - 3.2656i -1.4797	-1.2752 + 3.6665i -1.2752 - 3.6665i -1.4745

V. SIMULATION RESULTS

The original 4th order model is reduced to third order approximant and the dynamic response plots of step input are shown in Figs. 2(a)-2(d). To check the stability margins of the system closed loop system eigenvalues are obtained from the original and reduced order models and are shown in Table 1. In this work, a power system model is considered for the investigations. The single area power system model is reduced to the third order system from the original fourth order system . The dynamic system response plots are obtained for both power system original model and reduced order model. These dynamic response plots are obtained from various system states for considering a step disturbance in the system. From response plots it can be revealed that the dynamic response of the system states is better for the reduced order system as compared to those of the original system in case of power system model-1. Moreover, the system dynamic behaviour of reduced order as well as original system is comparable system in all respects.

VI. CONCLUSIONS

Routh approximation method is applied to an isolated power system model. The investigations of simulation results demonstrate that this method is computationally simple and always produces stable reduced model for stable original system. To validate the reduced order model, the dynamic behavior of original and reduced order model is achieved by applying a step disturbance in the system and is compared. Step responses shows that the characteristics of reduced model are comparable with that of original model. . There is no need to find eigenvalues and eigen vectors of the original system which simplifies the calculation to a great extent, while implementing Routh approximation method for model order reduction.

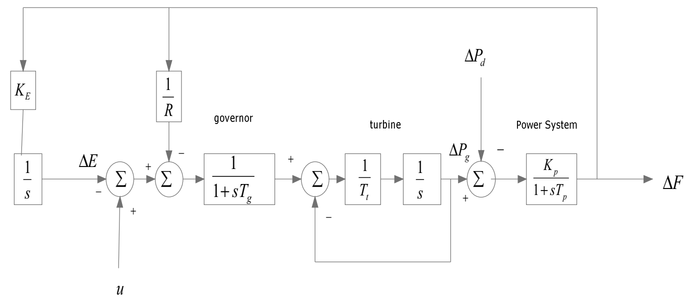

Fig.1. Transfer function model of AGC of a single area power system

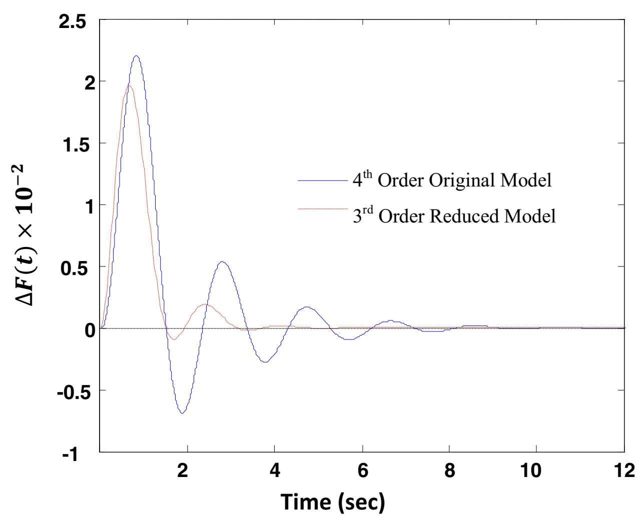

Fig. 2(a) Dynamic response of $\Delta F(t)$ for 1% load disturbance

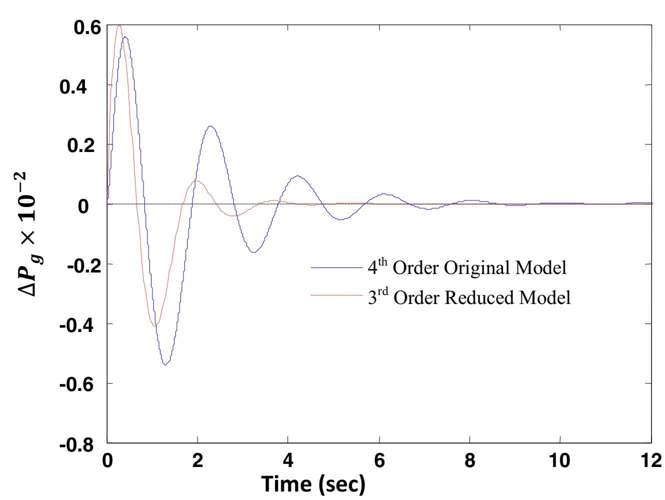

Fig. 2(b) Dynamic response of ΔP_g for 1% load disturbance

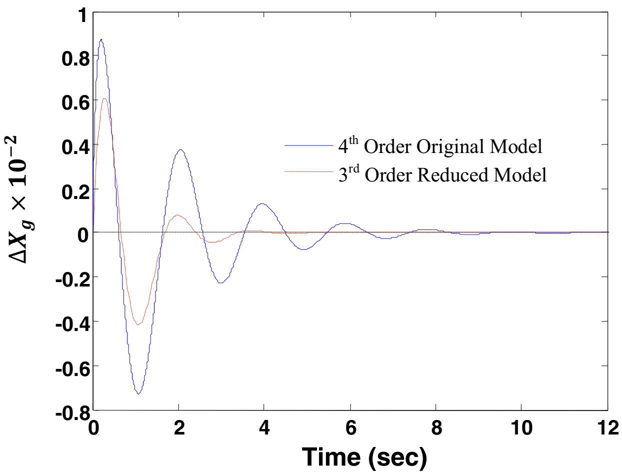

Fig.2 (c) Dynamic response of ΔX_g for 1% load disturbance

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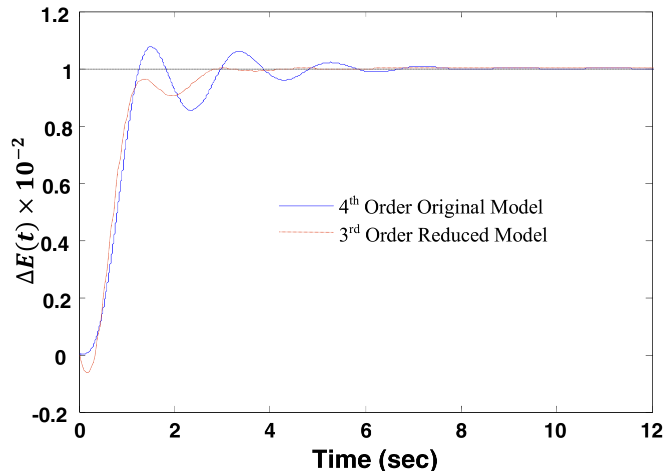

Fig.2 (d) Dynamic response of $\Delta E(t)$ for 1% load disturbance

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