International Conference on Innovative Applications of Computational Intelligence on Power, Energy and Controls with their Impact on Humanity (CIPECH14) 28 & 29 November 2014

Evaluating Fuzzy Reliability Using Rough Intuitionistic Fuzzy Set

Deepak Kumar
Department of Mathematics, Statistics & Computer Science,
G. B. Pant University of Agriculture & Technology
Pantnagar-263145, Uttarakhand, India.
deepakdev16@gmail.com

S. B. Singh
Department of Mathematics, Statistics & Computer Science,
G. B. Pant University of Agriculture & Technology
Pantnagar-263145, Uttarakhand, India.
drsurajbsingh@yahoo.com

***Abstract*—Fuzzy set theory has been studied extensively over the past several decades. It has been applied to solve many real life problems including system reliability. In this paper we define rough intuitionistic fuzzy sets with their properties and present a new method for analyzing fuzzy reliability of different types of systems considering the reliability of the components as rough intuitionistic fuzzy sets for handling the vagueness and incompleteness in information.**

Keywords— fuzzy system reliability; series and parallel network system; rough intuitionistic fuzzy set

I. INTRODUCTION

Fuzzy set that allow to manage imprecise and vague information was introduced by [11]. Fuzzy set theory has been widely and successfully applied in many different areas to handle vagueness. In fuzzy set only the membership degree is considered. The concept of intuitionistic fuzzy set was introduced [7], which allow to incorporate simultaneously the membership degree and the non-membership degree of each element, as a generalization of the notion of a fuzzy set. It is interesting and useful in modelling several real life problems. While fuzzy set is a powerful tool to deal with vagueness, the theory of rough sets introduced by [14] is a powerful mathematical tool to deal with incompleteness. Fuzzy sets and rough sets are two different concepts ([11] and [14]), none conflicts the other. Reference [3] defined rough fuzzy sets and fuzzy rough sets providing hints on some research directions on them. Reference [13], [1] and [15] also defined fuzzy rough sets independently in different ways. Reference [1], [3] and [11] defined rough fuzzy sets and fuzzy rough sets providing hints on some research directions on them. Rough fuzzy sets are concerned with both vagueness and incompleteness.

Fuzzy logic is becoming increasingly popular resulting in wide applications in several areas of real world including reliability theory. Reliability of an item is the probability that the item will perform a specified function under some specified condition for some specified time period. But in real life situation, it is not possible to always have accurate and complete information about the system. Therefore, in many cases it is very difficult to calculate the reliability of the system. To handle the insufficient and incomplete information, the rough fuzzy set theory approach can be used to estimate the reliability of system. The concept of fuzzy reliability has been proposed and developed by several authors [2], [4], [9] and [10]. These researches are based on possibility assumption or fuzzy state assumption. Recently, the researchers [2], [5], [6], [9] and [12] assumed that the reliability of each component is a fuzzy variable and investigated the fuzzy reliability of systems.

In the present paper we define rough intuitionistic fuzzy sets with their properties and present a new method for analyzing fuzzy reliability of different types of systems based on rough intuitionistic fuzzy set theory to handle the vagueness as well as incompleteness in information, where the reliabilities of components of a system are represented by rough intuitionistic fuzzy set. Finally some numerical examples are also presented to illustrate how to calculate the fuzzy system reliability using rough intuitionistic fuzzy set theory.

II. PRELIMINARIES

In this section we present some preliminaries which will be useful to our work in the next section

A. Definition:

Let *U* be any non-empty set and *R* is an equivalence relation over *U*. For any non-null subset *X* of *U*, the sets $A1(x)=\{x:[x]_R \subseteq X\}$ and $A2(X)=\{x:[x]_R \cap X \neq \phi\}$ are called the lower approximation and upper approximation, respectively of *X*, where the pair $S=(U,R)$ is called an approximation space. This equivalent relation *R* is called indiscernibility relation. The pair $A(x)=(A1(x),A2(x))$ is called the rough set of *X* in *S*. Here $[x]_R$ denotes the equivalence class of *R* containing *x*.

B. Definition:

Let $A=(A1,A2)$ and $B=(B1,B2)$ be two rough sets in the approximation space $S=(U,R)$. Then,

$$A\cup B=(A1\cup B1, A2\cup B2),\ A\cap B=(A1\cap B1, A2\cap B2),$$
$$A\subset B \text{ if } A\cap B=A,\ -A=\{U-A2, U-A1\}$$

For more details on the algebra and operations on rough sets one refer may [14].

C. Definition:

Let E be a fixed universe. An IFS A in E is an object having the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in E\}$ where the functions $\mu_A : E\to[0,1]$ and $\nu_A : E\to[0,1]$ define the degree of membership and the degree of non-membership respectively of the element $x\in E$ to the set A, and $\forall x\in E$

$$0\le\mu_A(x)+\nu_A(x)\ge 1.$$

$$\mu_F(x)\le 1-\nu_F(x)$$

$$\sup\{\mu_F(x)|x\in X_i\}\le\sup\{1-\nu_F(x)|x\in X_i\}$$

$$\sup\{\mu_F(x)|x\in X_i\}\le 1-\inf\{\nu_F(x)|x\in X_i\}$$

$$\sup\{\mu_F(x)|x\in X_i\}+\inf\{\nu_F(x)|x\in X_i\}\le 1$$

Fuzzy sets can be viewed as intuitionistic fuzzy sets but not conversely [7]. For various operations and relations on IFSs, one can refer [7] and [8].

III. ROUGH INTUITIONISTIC FUZZY SETS

The notion of rough fuzzy sets has been introduced by [3], giving few applications. In this section we define rough intuitionistic fuzzy sets and some operations viz. union, intersection inclusion and equalities over them.

A. Definition:

Let X be a non-null set and R be an equivalence relation on X. Let F be an intuitionistic fuzzy set (IFS) in X with the membership function μ_F and non-membership function ν_F. The lower and the upper approximations *R1(F)* and *R2(F)* respectively of the intuitionistic fuzzy set F are intuitionistic fuzzy sets of the quotient set X/R with

Membership function defined by

$$\mu_{R1(F)}(X_i)=\inf\{\mu_F(x):x\in X_i\},\mu_{R2(F)}(X_i)=\sup\{\mu_F(x):x\in X_i\}$$

Non -membership function defined by

$$\nu_{R1(F)}(X_i)=\sup\{\nu_F(x):x\in X_i\},\nu_{R2(F)}(X_i)=\inf\{\nu_F(x):x\in X_i\}$$

We prove that *R1(F)* and *R2(F)* defined in this way are IFS.
For $x\in X_i$, we obtain successively

$$\mu_F(x)+\nu_F(x)\le 1$$

$$\mu_F(x)\le 1-\nu_F(x)$$

$$\inf\{\mu_F(x)|x\in X_i\}\le\inf\{1-\nu_F(x)|x\in X_i\}$$

$$\inf\{\mu_F(x)|x\in X_i\}\le 1-\sup\{\nu_F(x)|x\in X_i\}$$

$$\inf\{\mu_F(x)|x\in X_i\}+\sup\{\nu_F(x)|x\in X_i\}\le 1$$

Hence *R1(F)* is an IFS. Similarly we can prove that *R2(F)* is also an IFS. The rough intuitionistic fuzzy set of F denoted by *R(F)* is given by the pair

$$R(F)=\langle R1(F),R2(F)\rangle$$

B. Definition:

If $R(F)=\langle R1(F),R2(F)\rangle$ is a rough intuitionistic fuzzy set F in *(X,R)*, the rough complement of *R(F)* is the rough intuitionistic fuzzy set denoted by $-R(F)$ and is defined by $-R(F)=\langle (R2(F))^c,(R1(F))^c\rangle$ where $(R2(F))^c$, $(R1(F))^c$ are the complements of the intuitionistic fuzzy sets *R2(F)* and *R1(F)* respectively.

C. Definition:

If $R(F_1)$ and $R(F_2)$ are two rough intuitionistic fuzzy sets of the intuitionistic fuzzy sets F_1 and F_2 respectively in X, then we define the following

(i) $R(F_1)=R(F_2)$ iff $R1(F_1)=R1(F_2)$ and $R2(F_1)=R2(F_2)$

(ii) $R(F_1)\subseteq R(F_2)$ iff $R1(F_1)\subseteq R1(F_2)$ and $R2(F_1)\subseteq R2(F_2)$

(iii) $R(F_1)\cup R(F_2)=\langle R1(F_1)\cup R1(F_2),R2(F_1)\cup R2(F_2)\rangle$

(iv) $R(F_1)\cap R(F_2)=\langle R1(F_1)\cap R1(F_2),R2(F_1)\cap R2(F_2)\rangle$

(v) $R(F_1)+R(F_2)=\langle R1(F_1)+R1(F_2),R2(F_1)+R2(F_2)\rangle$

(vi) $R(F_1).R(F_2)=\langle R1(F_1).R1(F_2),R2(F_1).R2(F_2)\rangle$

IV. FUZZY SYSTEM RELIABILITY EVALUATION USING ROUGH INTUITIONISTIC FUZZY SET

Here we present some propositions based on the new method for analyzing fuzzy system reliability using Rough intuitionistic fuzzy set (Rough IFS) theory. Here the reliabilities of components of a system are represented by Rough IFS.

A. Proposition

If reliability of each component of series system taken as Rough IFS for handling the vagueness and incompleteness in information then the fuzzy reliability of series system (R_s) is given by

$$R_s=\left[\left\{\mu_{11}\mu_{21}...\mu_{n1},\sum_{i=1}^{n}\nu_{i1}-\sum_{i,j=1}^{n}\nu_{i1}\nu_{j1}+\sum_{i,j,k=1}^{n}\nu_{i1}\nu_{j1}\nu_{k1}...+(-1)^{n-1}\nu_{11}\nu_{21}...\nu_{n1}\right\},\right.$$
$$\left.\left\{\mu_{12}\mu_{22}...\mu_{n2},\sum_{i=1}^{n}\nu_{i2}-\sum_{i,j=1}^{n}\nu_{i2}\nu_{j2}+\sum_{i,j,k=1}^{n}\nu_{i2}\nu_{j2}\nu_{k2}...+(-1)^{n-1}\nu_{12}\nu_{22}...\nu_{n2}\right\}\right]$$

Proof: Let n components are connected in series as shown in Fig. 1.

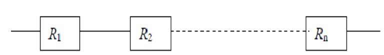

Fig. 1. Series System

Let $R_i=(R_{i1},R_{i2})$ be the reliability of the i^{th} component taken as Rough IFS where lower approximation is $R_{i1}=(\mu_{i1},\nu_{i1})$ and upper approximation is $R_{i2}=(\mu_{i2},\nu_{i2})$. Here μ_{i1} and ν_{i1} are membership and non-membership function of lower approximation of i^{th} unit respectively and μ_{i2} and ν_{i2} are membership and non-membership function of upper approximation of i^{th} unit respectively. Then, the reliability R_S of the system can be calculated as

$$R_s = \underset{i=1}{\overset{n}{\otimes}} R_i = R_1 \otimes R_2 \otimes \ldots\ldots \otimes R_n$$
$$= (R_{11}, R_{12}) \otimes (R_{21}, R_{22}) \otimes \ldots\ldots \otimes (R_{n1}, R_{n2})$$
$$= \{(R_{11} \otimes R_{21} \otimes \ldots \otimes R_{n1}), (R_{12} \otimes R_{22} \otimes \ldots \otimes R_{n2})\}$$
$$= [\{(\mu_{11}, \nu_{11}) \otimes (\mu_{21}, \nu_{21}) \otimes \ldots . \otimes (\mu_{n1}, \nu_{n1})\},$$
$$\{(\mu_{12}, \nu_{12}) \otimes (\mu_{22}, \nu_{22}) \otimes \ldots . \otimes (\mu_{n2}, \nu_{n2})\}]$$
$$= \left[\left\{\mu_{11}\mu_{21} \ldots \mu_{n1}, \sum_{i=1}^{n} \nu_{i1} - \sum_{i,j=1}^{n} \nu_{i1}\nu_{j1} + \sum_{i,j,k=1}^{n} \nu_{i1}\nu_{j1}\nu_{k1} \ldots \right.\right.$$
$$\left. + (-1)^{n-1} \nu_{11}\nu_{21} \ldots \nu_{n1}\right\}, \left\{\mu_{12}\mu_{22} \ldots \mu_{n2}, \sum_{i=1}^{n} \nu_{i2} - \sum_{i,j=1}^{n} \nu_{i2}\nu_{j2}\right.$$
$$\left.\left. + \sum_{i,j,k=1}^{n} \nu_{i2}\nu_{j2}\nu_{k2} \ldots + (-1)^{n-1} \nu_{12}\nu_{22} \ldots \nu_{n2}\right\}\right]$$

B. Proposition

If reliability of each component of parallel system taken as Rough IFS for handling the vagueness and incompleteness in information then the fuzzy reliability of parallel system (R_P) is given by $R_p = [(1-U_2, 1-V_2), (1-U_1, 1-V_1)]$ where

$U_1 = p_{12}p_{22} \ldots p_{n2}$,

$V_1 = \sum_{i=1}^{n} q_{i2} - \sum_{i,j=1}^{n} q_{i2}q_{j2} + \sum_{i,j,k=1}^{n} q_{i2}q_{j2}q_{k2} \ldots + (-1)^{n-1} q_{12}q_{22} \ldots q_{n2}$,

$U_2 = p_{11}p_{21} \ldots p_{n1}$ and

$V_2 = \sum_{i=1}^{n} q_{i1} - \sum_{i,j=1}^{n} q_{i1}q_{j1} + \sum_{i,j,k=1}^{n} q_{i1}q_{j1}q_{k1} \ldots + (-1)^{n-1} q_{11}q_{21} \ldots q_{n1}$

$p_{ij} = 1 - \mu_{ij}$ and $q_{ij} = 1 - \nu_{ij}$

Proof: Let n components are connected in parallel as shown in Fig. 2.

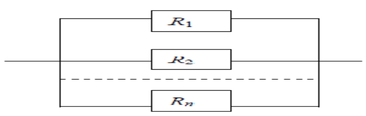

Fig. 2. Parallel System

If $R_i = (R_{i1}, R_{i2})$ be the reliability of the i^{th} component governed by Rough IFS where lower approximation is $R_{i1} = (\mu_{i1}, \nu_{i1})$ and upper approximation is $R_{i2} = (\mu_{i2}, \nu_{i2})$. Here μ_{i1} and ν_{i1} are membership and non-membership function of lower approximation of i^{th} unit respectively and μ_{i2} and ν_{i2} are membership and non-membership function of upper approximation of i^{th} component respectively. Then the reliability R_p of the system can be calculated as

$$R_p = \left(\underset{i=1}{\overset{n}{\otimes}} R_i^{\,c}\right)^c$$
$$R_p^{\,c} = \underset{i=1}{\overset{n}{\otimes}} R_i^{\,c}$$
$$R_p^{\,c} = R_1^{\,c} \otimes R_2^{\,c} \otimes \ldots\ldots \otimes R_n^{\,c}$$
$$R_p^{\,c} = (R_{11}, R_{12})^c \otimes (R_{21}, R_{22})^c \otimes \ldots\ldots \otimes (R_{n1}, R_{n2})^c$$
$$R_p^{\,c} = (R_{12}^{\,c}, R_{11}^{\,c}) \otimes (R_{22}^{\,c}, R_{21}^{\,c}) \otimes \ldots\ldots \otimes (R_{n2}^{\,c}, R_{n1}^{\,c})$$
$$R_p^{\,c} = \{(R_{12}^{\,c} \otimes R_{22}^{\,c} \otimes \ldots \otimes R_{n2}^{\,c}), (R_{11}^{\,c} \otimes R_{21}^{\,c} \otimes \ldots \otimes R_{n1}^{\,c})\}$$
$$R_{ij} = (\mu_{ij}, \nu_{ij})$$
$$R_{ij}^{\,c} = (1-\mu_{ij}, 1-\nu_{ij})$$
$$R_{ij}^{\,c} = (p_{ij}, q_{ij}) \text{ where } p_{ij} = 1-\mu_{ij} \text{ and } q_{ij} = 1-\nu_{ij}$$
$$R_p^{\,c} = [\{(p_{12}, q_{12}) \otimes (p_{22}, q_{22}) \otimes \ldots \otimes (p_{n2}, q_{n2})\},$$
$$\{(p_{11}, q_{11}) \otimes (p_{21}, q_{21}) \otimes \ldots \otimes (p_{n1}, q_{n1})\}]$$
$$R_p^{\,c} = \left[\left\{p_{12}p_{22} \ldots p_{n2}, \sum_{i=1}^{n} q_{i2} - \sum_{i,j=1}^{n} q_{i2}q_{j2} + \sum_{i,j,k=1}^{n} q_{i2}q_{j2}q_{k2} \ldots \right.\right.$$
$$\left. + (-1)^{n-1} q_{12}q_{22} \ldots q_{n2}\right\}, \left\{p_{11}p_{21} \ldots p_{n1}, \sum_{i=1}^{n} q_{i1} - \sum_{i,j=1}^{n} q_{i1}q_{j1}\right.$$
$$\left.\left. + \sum_{i,j,k=1}^{n} q_{i1}q_{j1}q_{k1} \ldots + (-1)^{n-1} q_{11}q_{21} \ldots q_{n1}\right\}\right]$$
$$R_p^{\,c} = [(U_1, V_1), (U_2, V_2)]$$

where

$U_1 = p_{12}p_{22} \ldots p_{n2}$,

$V_1 = \sum_{i=1}^{n} q_{i2} - \sum_{i,j=1}^{n} q_{i2}q_{j2} + \sum_{i,j,k=1}^{n} q_{i2}q_{j2}q_{k2} \ldots + (-1)^{n-1} q_{12}q_{22} \ldots q_{n2}$,

$U_2 = p_{11}p_{21} \ldots p_{n1}$ and

$V_2 = \sum_{i=1}^{n} q_{i1} - \sum_{i,j=1}^{n} q_{i1}q_{j1} + \sum_{i,j,k=1}^{n} q_{i1}q_{j1}q_{k1} \ldots + (-1)^{n-1} q_{11}q_{21} \ldots q_{n1}$

$$R_p = [(U_2, V_2)^c, (U_1, V_1)^c]$$
$$R_p = [(1-U_2, 1-V_2), (1-U_1, 1-V_1)]$$

C. Proposition

If reliability of each component of parallel-series system taken as Rough IFS to handle subjectivity, ambiguity, uncertainty and imprecision then the fuzzy reliability of parallel-series system (Rps) is given by

$$R_{ps}^{\,c} = \underset{i=1}{\overset{n}{\otimes}} \left\langle \left[\underset{j=1}{\overset{m}{\otimes}} R_{i2}^{j}\right]^c, \left[\underset{j=1}{\overset{m}{\otimes}} R_{i1}^{j}\right]^c \right\rangle$$

where

$$\underset{j=1}{\overset{m}{\otimes}} R_{i1}^{j} = \left[\mu_{i1}^{1}\mu_{i1}^{2} \ldots \mu_{i1}^{m}, \sum_{j=1}^{m} \nu_{i1}^{j} - \sum_{j,k=1}^{m} \nu_{i1}^{j}\nu_{i1}^{k} + \sum_{j,k,l=1}^{m} \nu_{i1}^{j}\nu_{i1}^{k}\nu_{i1}^{l} \ldots + (-1)^{m-1} \nu_{i1}^{1}\nu_{i1}^{2} \ldots \nu_{i1}^{m}\right]$$

and

$$\underset{j=1}{\overset{m}{\otimes}} R_{i2}^{j} = \left[\mu_{i2}^{1}\mu_{i2}^{2} \ldots \mu_{i2}^{m}, \sum_{j=1}^{m} \nu_{i2}^{j} - \sum_{j,k=1}^{m} \nu_{i2}^{j}\nu_{i2}^{k} + \sum_{j,k,l=1}^{m} \nu_{i2}^{j}\nu_{i2}^{k}\nu_{i2}^{l} \ldots + (-1)^{m-1} \nu_{i2}^{1}\nu_{i2}^{2} \ldots \nu_{i2}^{m}\right]$$

Proof: Consider a parallel-series system consists of n subsystems which are connected in parallel and each subsystem contains m components as shown in Fig. 3.

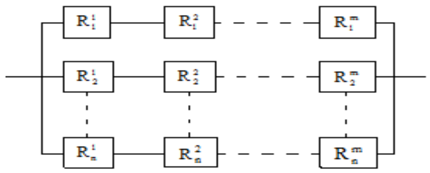

Fig. 3. Parallel-Series System

If $R_i^j = \left(R_{i1}^j, R_{i2}^j\right)$ be the reliability of the i^{th} component taken as Rough IFS where lower approximation is $R_{i1}^j = \left(\mu_{i1}^j, \nu_{i1}^j\right)$ and upper approximation is $R_{i2}^j = \left(\mu_{i2}^j, \nu_{i2}^j\right)$. Here μ_{i1}^j and ν_{i1}^j are membership and non-membership function of lower approximation of i^{th} component respectively and μ_{i2}^j and ν_{i2}^j are membership and non-membership function of upper approximation of i^{th} component respectively. Then the reliability R_{ps} of the system can be calculated as

$$R_{ps}{}^c = \underset{i=1}{\overset{n}{\otimes}} \left\langle \left[\underset{j=1}{\overset{m}{\otimes}} R_{i2}^j \right]^c, \left[\underset{j=1}{\overset{m}{\otimes}} R_{i1}^j \right]^c \right\rangle$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i1}^j = R_{i1}^1 \otimes R_{i1}^2 \otimes \ldots \otimes R_{i1}^m$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i1}^j = \left(\mu_{i1}^1, \nu_{i1}^1\right) \otimes \left(\mu_{i1}^2, \nu_{i1}^2\right) \otimes \ldots. \otimes \left(\mu_{i1}^m, \nu_{i1}^m\right)$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i1}^j = \left[\mu_{i1}^1 \mu_{i1}^2 \ldots \mu_{i1}^m, \sum_{j=1}^{m} \nu_{i1}^j - \sum_{j,k=1}^{m} \nu_{i1}^j \nu_{i1}^k + \sum_{j,k,l=1}^{m} \nu_{i1}^j \nu_{i1}^k \nu_{i1}^l \ldots + (-1)^{m-1} \nu_{i1}^1 \nu_{i1}^2 \ldots \nu_{i1}^m \right]$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i2}^j = R_{i2}^1 \otimes R_{i2}^2 \otimes \ldots \otimes R_{i2}^m$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i2}^j = \left(\mu_{i2}^1, \nu_{i2}^1\right) \otimes \left(\mu_{i2}^2, \nu_{i2}^2\right) \otimes \ldots. \otimes \left(\mu_{i2}^m, \nu_{i2}^m\right)$$

$$\underset{j=1}{\overset{m}{\otimes}} R_{i2}^j = \left[\mu_{i2}^1 \mu_{i2}^2 \ldots \mu_{i2}^m, \sum_{j=1}^{m} \nu_{i2}^j - \sum_{j,k=1}^{m} \nu_{i2}^j \nu_{i2}^k + + \sum_{j,k,l=1}^{m} \nu_{i2}^j \nu_{i2}^k \nu_{i2}^l \ldots + (-1)^{m-1} \nu_{i2}^1 \nu_{i2}^2 \ldots \nu_{i2}^m \right]$$

D. Proposition

If reliability of each component of series-parallel system taken as Rough IFS for handling the subjectivity, ambiguity, uncertainty and imprecision in information then the fuzzy reliability of series-parallel system (R_{sp}) is given by

$$R_{sp} = \underset{j=1}{\overset{m}{\otimes}} \left\langle \left[\underset{i=1}{\overset{n}{\otimes}} R_{i2}^j \right]^c, \left[\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j \right]^c \right\rangle$$

where

$$\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j = \left[\mu_{11}^j \mu_{21}^j \ldots \mu_{n1}^j, \sum_{i=1}^{n} \nu_{i1}^j - \sum_{i,k=1}^{n} \nu_{i1}^j \nu_{k1}^j + \sum_{i,k,l=1}^{n} \nu_{i1}^j \nu_{k1}^j \nu_{l1}^j \ldots + (-1)^{n-1} \nu_{11}^j \nu_{21}^j \ldots \nu_{n1}^j \right]$$

and

$$\underset{i=1}{\overset{n}{\otimes}} R_{i2}^j = \left[\mu_{12}^j \mu_{22}^j \ldots \mu_{n2}^j, \sum_{i=1}^{n} \nu_{i2}^j - \sum_{i,k=1}^{n} \nu_{i2}^j \nu_{k2}^j + \sum_{i,k,l=1}^{n} \nu_{i2}^j \nu_{k2}^j \nu_{l2}^j \ldots + (-1)^{n-1} \nu_{12}^j \nu_{22}^j \ldots \nu_{n2}^j \right]$$

Proof: Consider a series-parallel system consists of m subsystems which are connected in series and each subsystem contains n components as shown in Fig. 4.

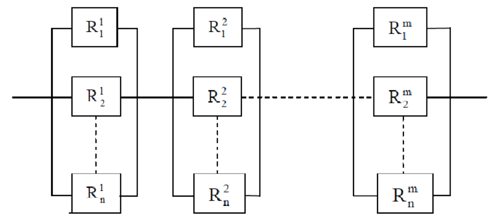

Fig. 4. Parallel-Series System

If $R_i^j = \left(R_{i1}^j, R_{i2}^j\right)$ be the reliability of the i^{th} component taken as Rough IFS where lower approximation is $R_{i1}^j = \left(\mu_{i1}^j, \nu_{i1}^j\right)$ and upper approximation is $R_{i2}^j = \left(\mu_{i2}^j, \nu_{i2}^j\right)$. Here μ_{i1}^j and ν_{i1}^j are membership and non-membership function of lower approximation of i^{th} component respectively and μ_{i2}^j and ν_{i2}^j are membership and non-membership function of upper approximation of i^{th} component respectively. Then the reliability R_{sp} of the system can be calculated as

$$R_{sp} = \underset{j=1}{\overset{m}{\otimes}} \left[\underset{i=1}{\overset{n}{\otimes}} R_i^j \right]^c$$

$$R_{sp} = \underset{j=1}{\overset{m}{\otimes}} \left\langle \left[\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j, \underset{i=1}{\overset{n}{\otimes}} R_{i2}^j \right] \right\rangle^c$$

$$R_{sp} = \underset{j=1}{\overset{m}{\otimes}} \left\langle \left[\underset{i=1}{\overset{n}{\otimes}} R_{i2}^j \right]^c, \left[\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j \right]^c \right\rangle$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j = R_{11}^j \otimes R_{21}^j \otimes \ldots \otimes R_{n1}^j$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j = \left(\mu_{11}^j, \nu_{11}^j\right) \otimes \left(\mu_{21}^j, \nu_{21}^j\right) \otimes \ldots. \otimes \left(\mu_{n1}^j, \nu_{n1}^j\right)$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i1}^j = \left[\mu_{11}^j \mu_{21}^j \ldots \mu_{n1}^j, \sum_{i=1}^{n} \nu_{i1}^j - \sum_{i,k=1}^{n} \nu_{i1}^j \nu_{k1}^j + \sum_{i,k,l=1}^{n} \nu_{i1}^j \nu_{k1}^j \nu_{l1}^j \ldots + (-1)^{n-1} \nu_{11}^j \nu_{21}^j \ldots \nu_{n1}^j \right]$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i2}^j = R_{12}^j \otimes R_{22}^j \otimes \ldots \otimes R_{n2}^j$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i2}^j = \left(\mu_{12}^j, \nu_{12}^j\right) \otimes \left(\mu_{22}^j, \nu_{22}^j\right) \otimes \ldots. \otimes \left(\mu_{n2}^j, \nu_{n2}^j\right)$$

$$\underset{i=1}{\overset{n}{\otimes}} R_{i2}^{j} = \left[\mu_{12}^{j} \mu_{22}^{j} \ldots \mu_{n2}^{j}, \sum_{i=1}^{n} \nu_{i2}^{j} - \sum_{i,k=1}^{n} \nu_{i2}^{j} \nu_{k2}^{j} + \sum_{i,k,l=1}^{n} \nu_{i2}^{j} \nu_{k2}^{j} \nu_{l2}^{j} \ldots + (-1)^{n-1} \nu_{12}^{j} \nu_{22}^{j} \ldots \nu_{n2}^{j} \right]$$

V. EXAMPLES

In this section some examples are illustrated to demonstrate prepositions discussed in section IV. Let

$R_1 = \{R_{11}, R_{12}\} = \{(\mu_{11}, \nu_{11}), (\mu_{12}, \nu_{12})\} = \{(0.6, 0.3), (0.8, 0.2)\}$,

$R_2 = \{R_{21}, R_{22}\} = \{(\mu_{21}, \nu_{21}), (\mu_{22}, \nu_{22})\} = \{(0.4, 0.6), (0.5, 0.5)\}$

and $R_3 = \{R_{31}, R_{32}\} = \{(\mu_{31}, \nu_{31}), (\mu_{32}, \nu_{32})\} = \{(0.1, 0.9), (0.3, 0.7)\}$

are the reliabilities (considered as a rough intuitionistic fuzzy set) of the three components used in the following examples.

A. Series System

Consider a series system containing three components then reliability of the system with the help of proposition *A* is given by

$$R_s = \underset{i=1}{\overset{3}{\otimes}} R_i = R_1 \otimes R_2 \otimes R_3$$
$$= \{(0.6,0.3), (0.8,0.2)\} \otimes \{(0.4,0.6), (0.5,0.5)\} \otimes \{(0.1,0.9), (0.3, 0.7)\}$$
$$= \{(0.024, 0.972), (0.12, 0.88)\}$$

B. Parallel System

Consider a parallel system containing three components then reliability of the system with the help of proposition *B* is given by

$$R_p = \left(R_1^{c} \otimes R_1^{c} \otimes R_1^{c} \right)^{c}$$
$$= \{(0.6,0.3), (0.8,0.2)\}^{c} \otimes \{(0.4,0.6), (0.5,0.5)\}^{c} \otimes \{(0.1,0.9), (0.3, 0.7)\}^{c}$$
$$= \{(0.784, 0.162), (0.93, 0.07)\}$$

C. Parallel-Series System

Let a parallel-series system consists of three subsystems which are connected in parallel and each subsystem contains three components then reliability of the system with the help of proposition *C* is given by

$$R_{ps}{}^{c} = \underset{i=1}{\overset{3}{\otimes}} \left[\underset{j=1}{\overset{3}{\otimes}} R_i^{j} \right]^{c}$$
$$R_{ps} = \{(0.070286, 0.91833), (0.318528, 0.681472)\}$$

D. Series-ParellelSytstem

Let a series-parallel system consists of three subsystems which are connected in series and each subsystem contains 3 components then reliability of the system with the help of proposition *D* is given by

$$R_{sp} = \underset{j=1}{\overset{3}{\otimes}} \left[\underset{i=1}{\overset{3}{\otimes}} R_i^{j} \right]^{c}$$
$$R_{sp} = \{(0.48189, 0.41152), (0.804357, 0.195643)\}$$

VI. CONCLUSION

In this paper we have presented a new method for analyzing fuzzy system reliability of different types of systems using rough intuitionistic fuzzy set theory, where the component of a system are represented by rough intuitionistic fuzzy set. The proposed method can model and analyze system reliability in a more flexible and more intelligent manner. The proposed methodology is a very promising mathematical approach for reliability modelling of systems, as it incorporates subjectivity, ambiguity, uncertainty and imprecision.

Results from the examples illustrated here also support the fact that the reliability is maximum in case when the components are connected in parallel configuration, while it is minimum in the case of series configuration.

REFERENCES

[1] A. Nakamura, "Fuzzy rough sets," Note on Multiple Valued Logic, vol. 9, pp. 1-8, August 1988.

[2] C. H. Cheng and D. L. Mon, "Fuzzy system reliability analysis by interval of confidence," Fuzzy Sets and Systems, vol. 56, pp. 29-35, May 1993.

[3] D. Dubois and H. prade, "Rough fuzzy sets and fuzzy rough sets," Int. Jou. Gen. Sys., vol. 17, pp. 191-209, March 1989.

[4] D. L. Mon and C. H. Cheng, "Fuzzy system reliability analysis for components with different membership functions," Fuzzy Sets and Systems, vol. 64, pp. 145-157, June 1994.

[5] D. L. Mon and C. H. Cheng, "A general and easy computational method for function of fuzzy numbers," Int. J. Systems Sci., vol. 26, pp. 983-990, November 1995.

[6] J. S. Wu and R. Chen, "An algorithm for computing the reliability of weighted-k-out-of-n systems," IEEE Trans. Reliab., vol. 43, pp. 327-328, September 1994.

[7] K. T. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, pp. 87-96, January 1986.

[8] K. T. Atanassov, "New operations defined over the Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 61, pp. 137-142, January 1994.

[9] K. Y. Cai, C. Y. Wen and M. L. Zhang, "Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context," Fuzzy Sets and Systems, vol. 42, pp. 145-172, December 1991.

[10] K. Y. Cai, C. Y. Wen and M. L. Zhang, "Posbist reliability behaviour of typical systems with two types of failures," Fuzzy Sets and Systems, vol. 43, pp. 17-32, March 1991.

[11] L. A. Zadeh, "Fuzzy sets. Information and Control," vol. 8, pp. 338-353, January 1965.

[12] L. V. Utkin and S. V. Gurov, "A general formal approach for fuzzy reliability analysis in the possibility context," Fuzzy Set and Systems, vol. 83, pp. 203-213, February 1995.

[13] S. Nanda and S. Majumdar, "Fuzzy rough sets," Fuzzy Sets and Systems, vol. 45, pp. 157-160, June 1992.

[14] Z. Pawlak, "Rough sets," Int. Jou. Info. Comp. Sc., vol. 11, pp. 341-356, June 1982.

[15] Z. Pawlak, "Rough sets and fuzzy sets," Fuzzy Sets and Systems, vol. 17, pp. 99-102, April 1985.